Human Population 2018

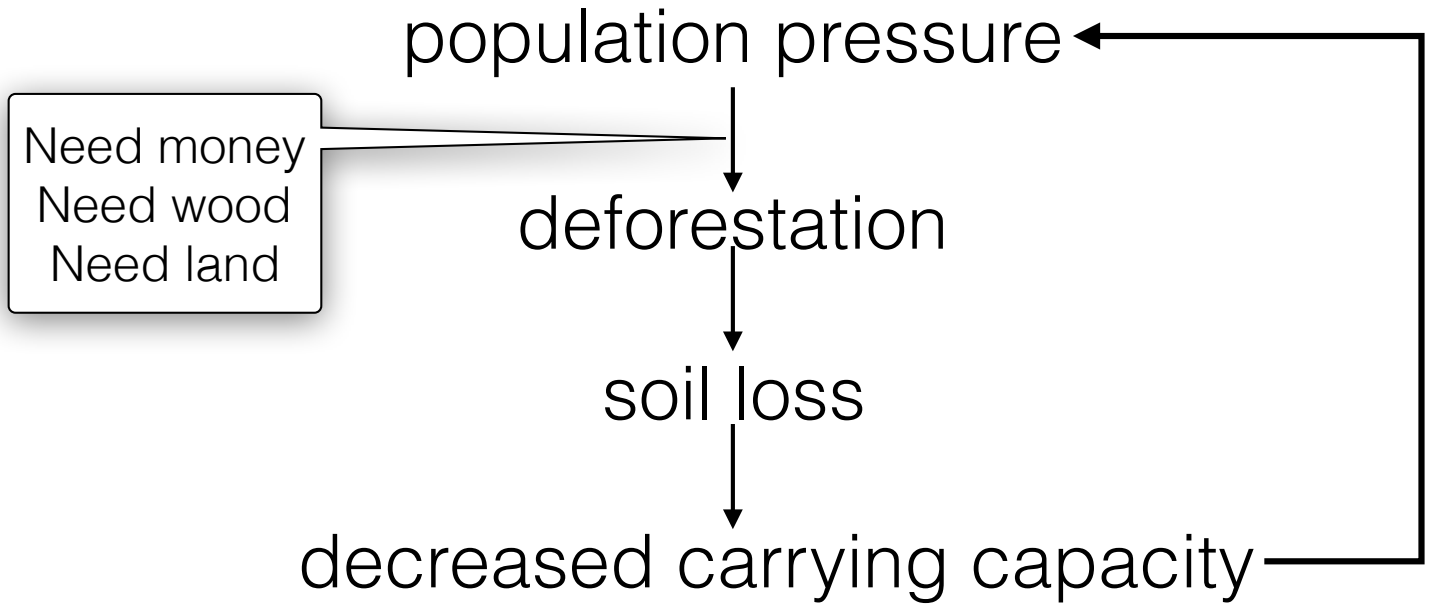
Lecture 16
modeling hyperexponential growth

Math and Modeling

Is there free will at the global scale?

Environmental determinism

- Determinism is the idea that the future is already decided "for us" by the Laws of Nature.



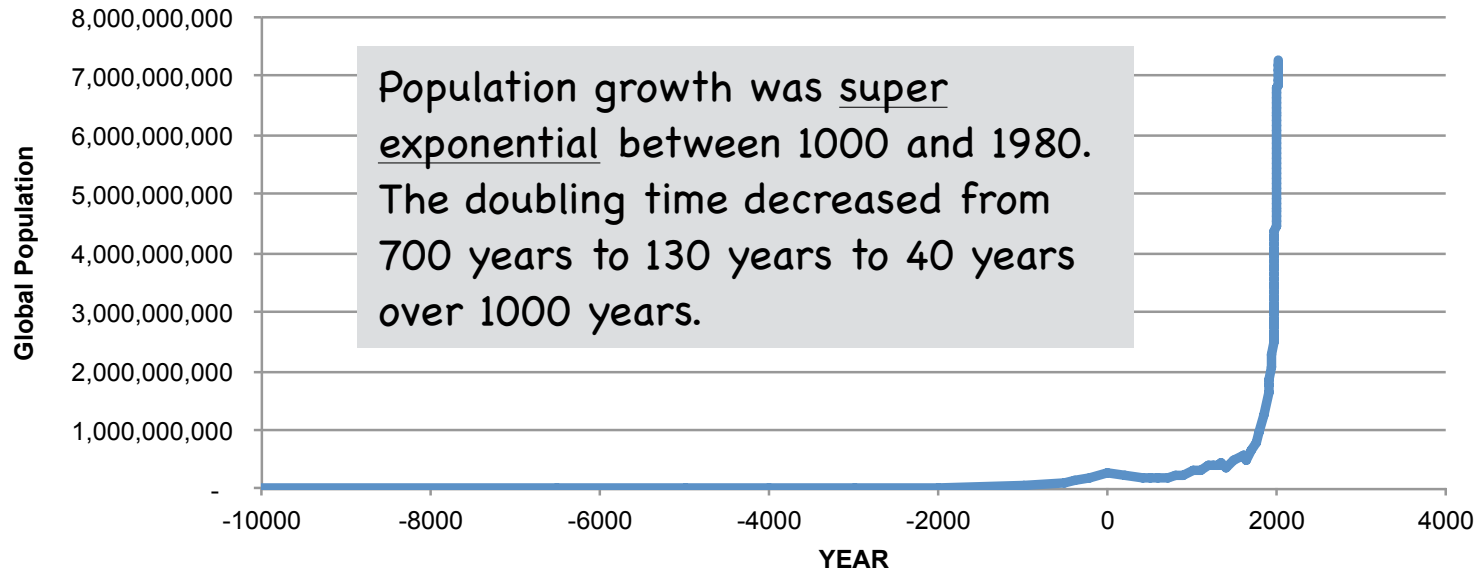
Does a deterministic model mean our future is pre-determined?

- No, because it is just a **model**.
- No, because the **variables** in the model are not all known.
- No, because
    - people don't always act as **expected**.
    - people change the **rules**.
    - people **adapt**.
- However …

  Determinism may be used as **an approach** to modeling that **sets a baseline**, over which are plotted **random events and unexpected actions**.
- Determinism in modeling means **we model behavior,** its output consequences and its input dependencies.

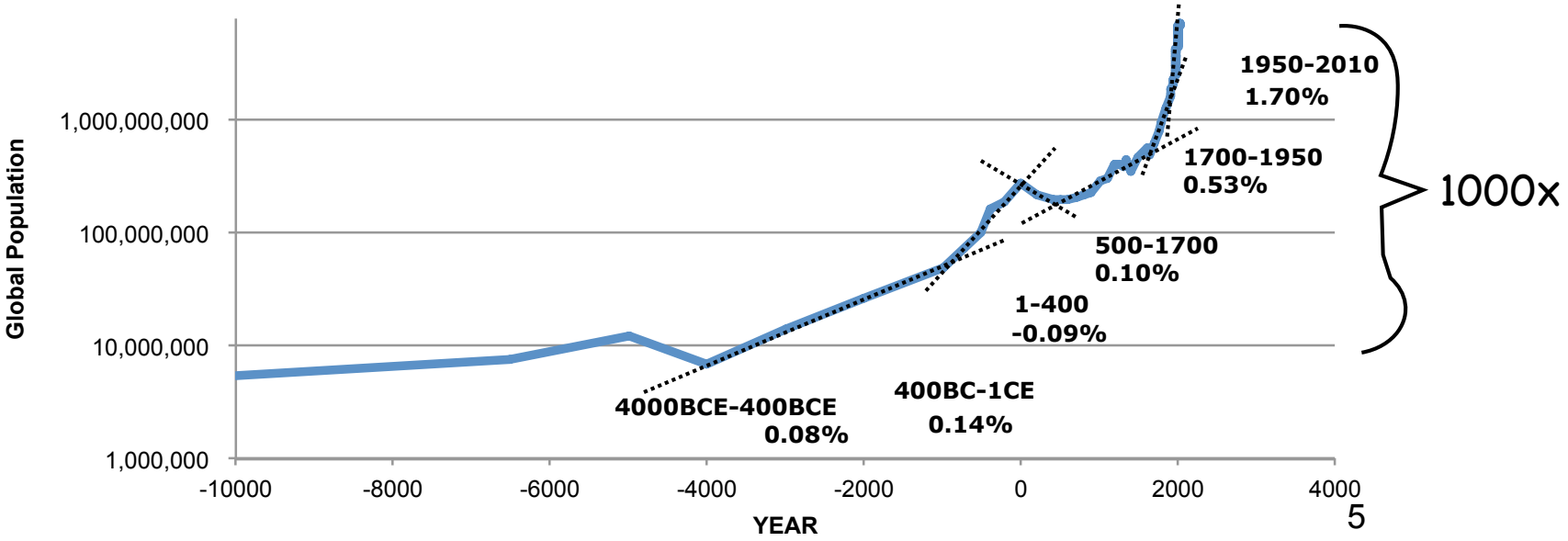
Human population since 10,000BC

linear scale



... does not fit a simple exponential growth curve.

log scale



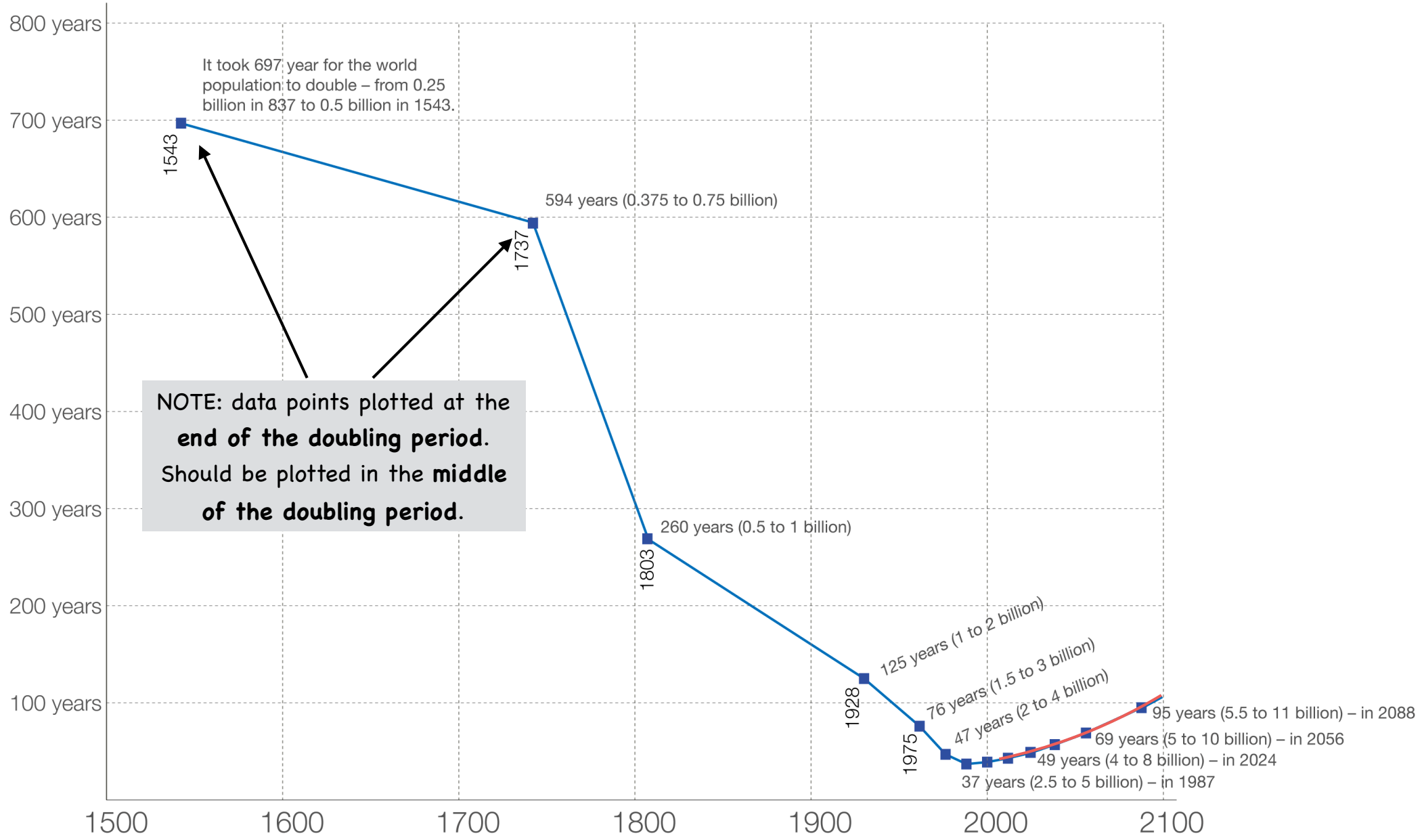
https://www.census.gov/programs-surveys/international-programs.html

The rate of exponential growth has accelerated.

Time it took for the world population to double

Historical estimates of the world population until 2015 – and UN projections until 2100

Our World in Data



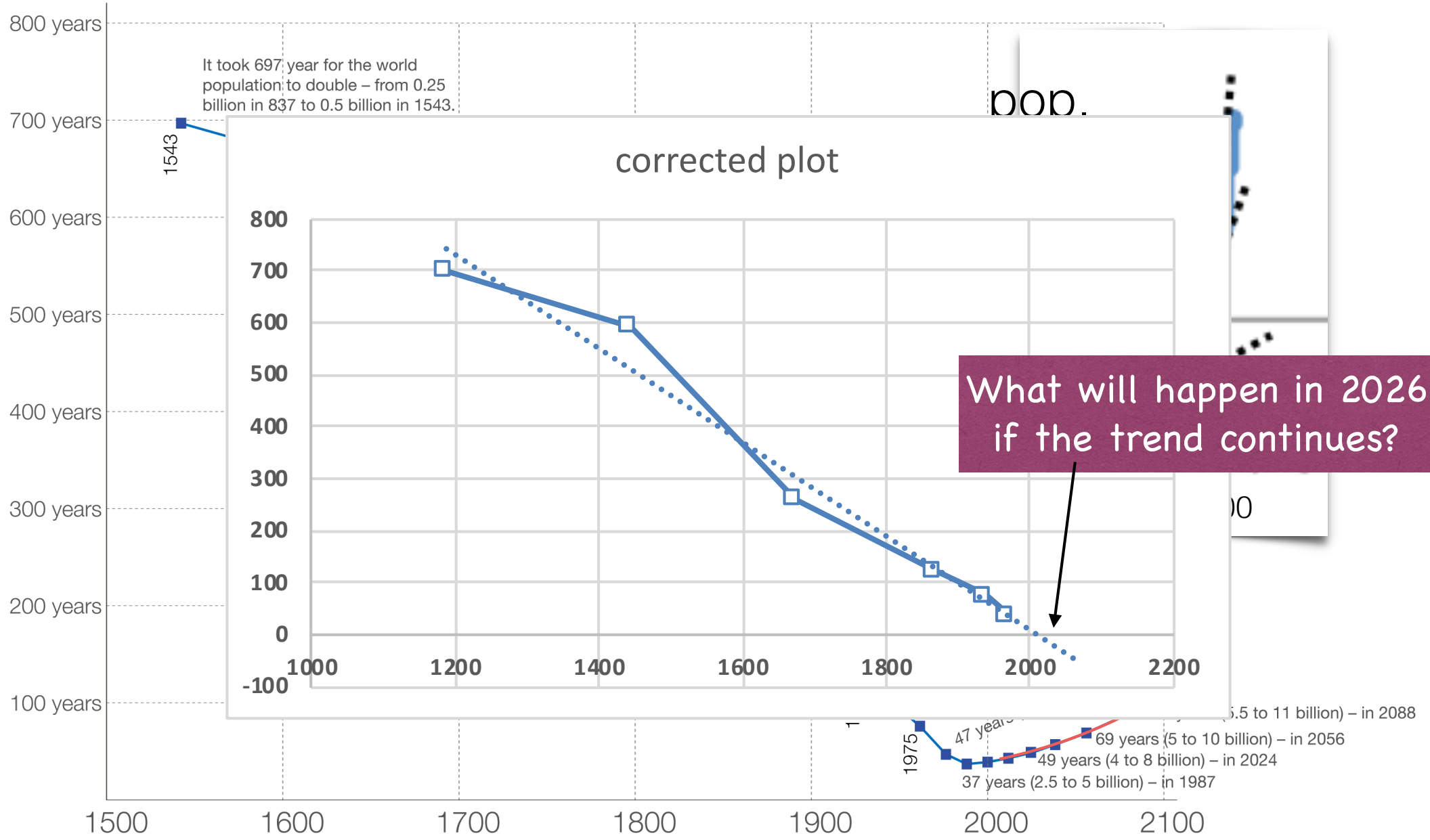
Data source: OurWorldInData annual world population series (Based on HYDE and UN until 2015. And projections from the UN after 2015 ('Medium Variant' 2015 Revision).
The data visualization is available at OurWorldinData.org. There you find the raw data, more visualizations, and research on this topic.

The doubling time is shrinking linearly.

Time it took for the world population to double

Historical estimates of the world population until 2015 – and UN projections until 2100

Our World in Data



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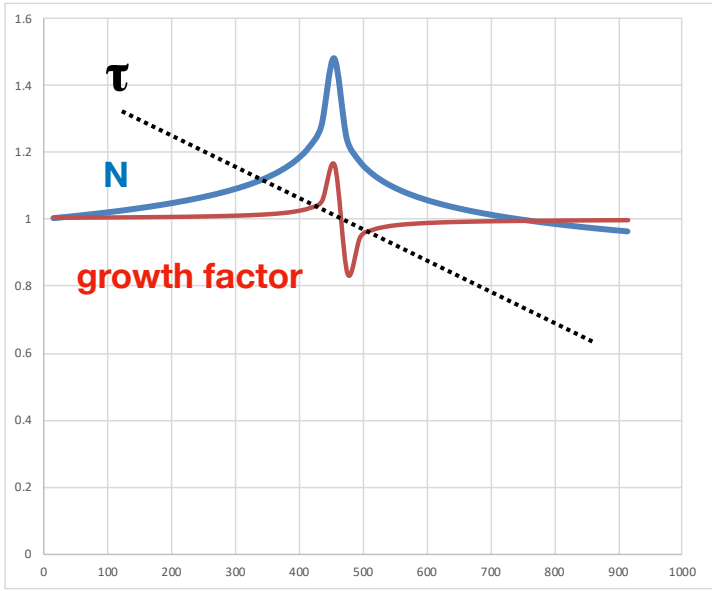
What does this trend mean? If anything. Is it predictive?

$\tau = \ln 2 / k$

$k = \ln(2)/\tau$

$\text{growth factor} \approx 1 + \ln(2)/\tau$

small negative τ means growth factor << 1

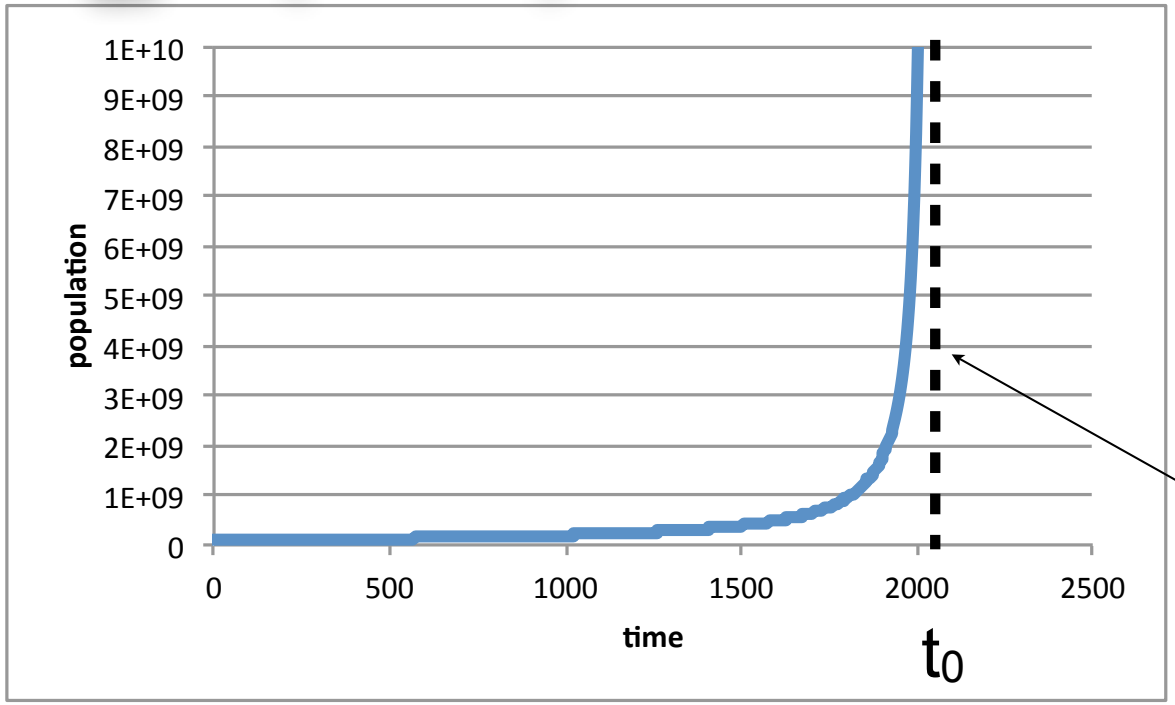


A linear trend in 1/k implies a singularity between intrinsic hyperbolic growth and intrinsic hyperbolic decline

A mathematical model for hyperexponential growth: Von Foerster's "Doomsday" growth equation

$$\frac{dN}{dt} = \left(\alpha_0 N^{1/k}\right) N$$

Growth *rate* is a function of N, not a constant.



Integrating yields…

$$N_t = N_1 \left(\frac{t_0 - t_1}{t_o - t}\right)^k$$

t_0 is doomsday, 13 Nov 2026

H. von Foerster, PM Mora, LW Amiot (1960) "Doomsday: Friday 13 November, A.D. 2026" Science132:1291-95.

Von Foerster's equation is hyperbolic

Standard exponential growth equation:

$$\frac{dN}{dt} = \alpha_0 N$$

Growth is proportional to the number of people, because births and deaths are both proportional to number of people.

Doomsday growth equation:

$$\frac{dN}{dt} = \left(\alpha_0 N^{1/k}\right) N$$

This additional term says that growth *rate* (the difference between birth rate and death rate) increases with population. This is not generally true for living organisms. *But it fits the data.*

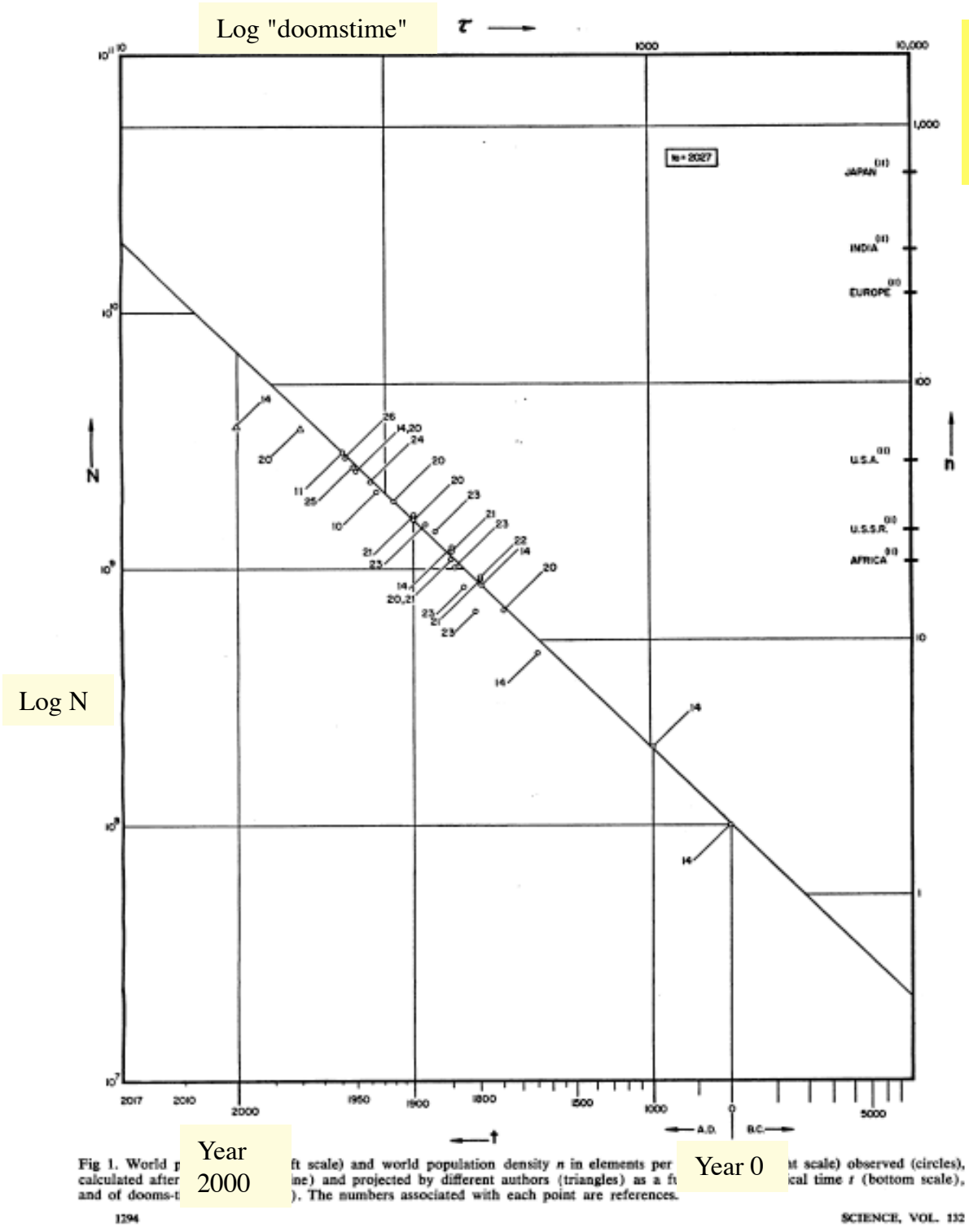


Fig 1. World p ft scale) and world population density n in elements per t scale) observed (circles), calculated after ine) and projected by different authors (triangles) as a ft ical time t (bottom scale), and of dooms-t). The numbers associated with each point are references.

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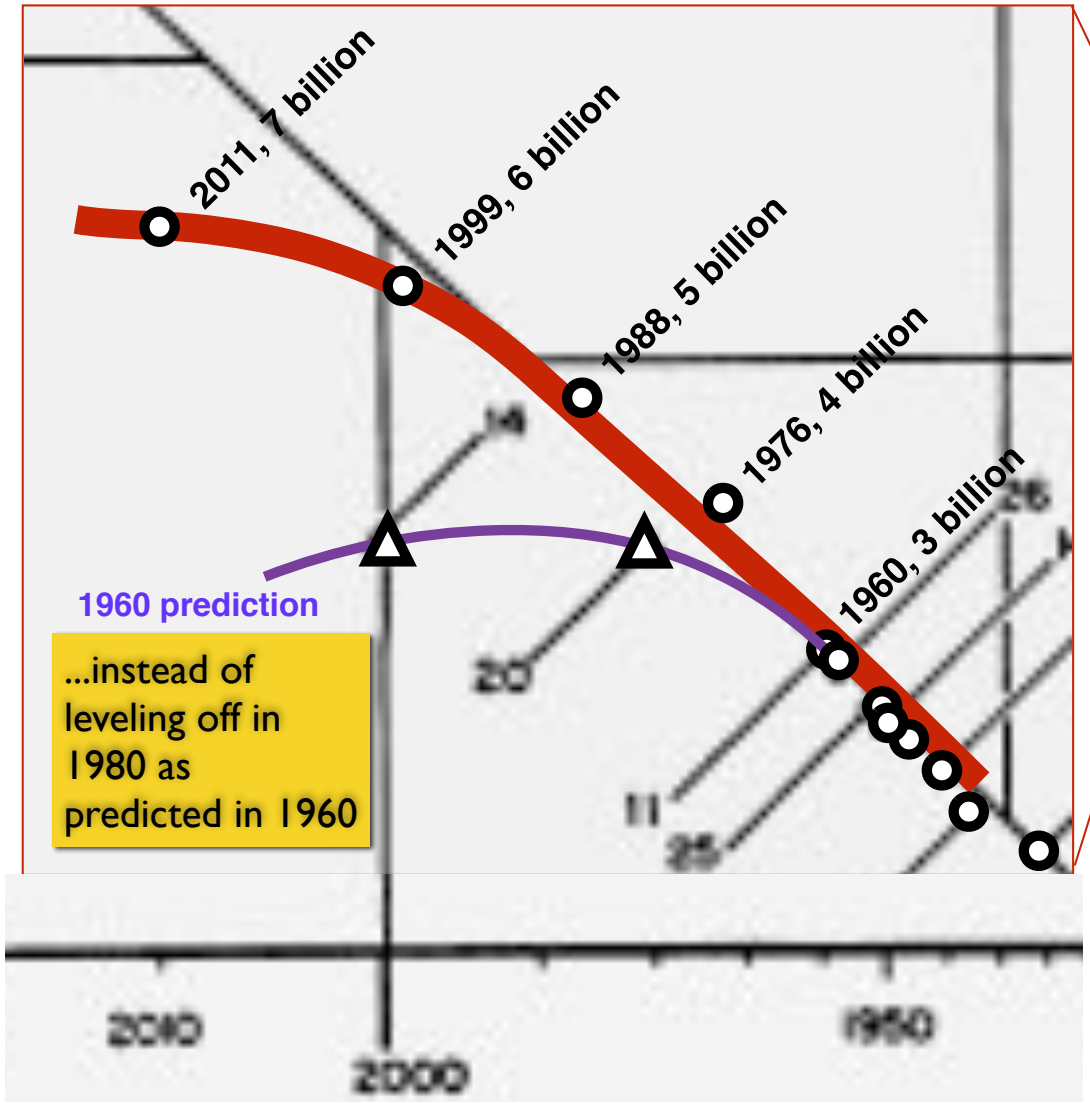
The Doomsday equation fits historical global population data up to 1960

$$N_t = N_1 \left(\frac{t_0 - t_1}{t_o - t} \right)^k$$

Log-log plot of historical human population versus time (lower x-axis) and dooms-time (upper x-axis). The fit is hyperbolic!

H. von Foerster, PM Mora, LW Amiot (1960) "Doomsday: Friday 13 November, A.D. 2026" Science132:1291-95.

Population growth since 1960 still tracked the Doomsday Equation until around 1999...



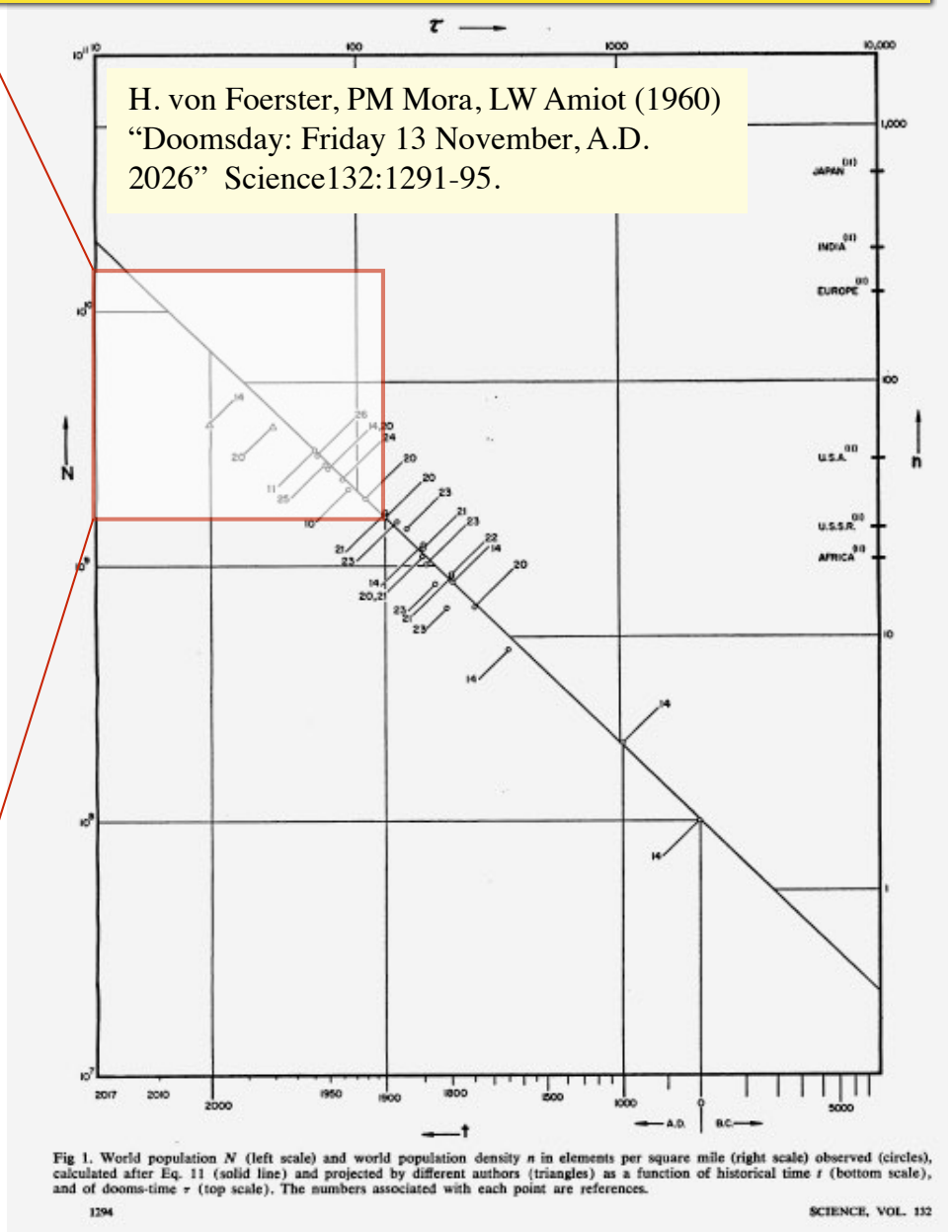


Fig 1. World population N (left scale) and world population density n in elements per square mile (right scale) observed (circles), calculated after Eq. 11 (solid line) and projected by different authors (triangles) as a function of historical time t (bottom scale), and of dooms-time τ (top scale). The numbers associated with each point are references.

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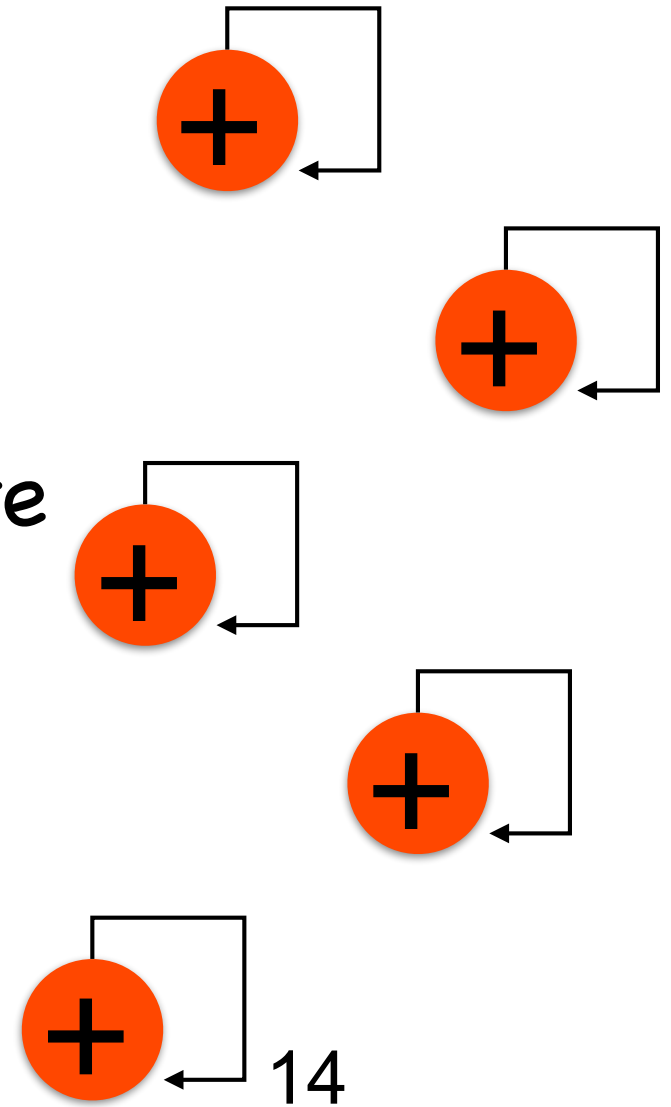
Von Foerster's Rationalle: increasing unity and cohesion in the human experience

However, what may be true for* elements which, because of lack of adequate communication among each other, have to resort to a competitive, (almost) zero-sum multiperson game may be false for elements that possess a system of communication which enables them to form coalitions until all elements are so strongly linked that the population as a whole can be considered from a game-theoretical point of view as a single person playing a two-person game with nature as its opponent. In this situation it is not absurd to assume that an increase in elements may produce a more versatile and effective coalition and thus not only may render environmental hazards less effective but also may improve the living conditions beyond those found in a "natural setting."

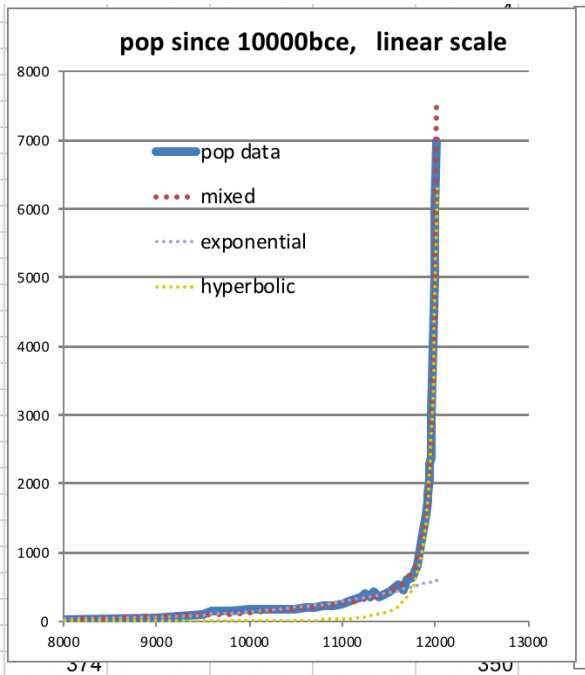
*"elements" are individuals

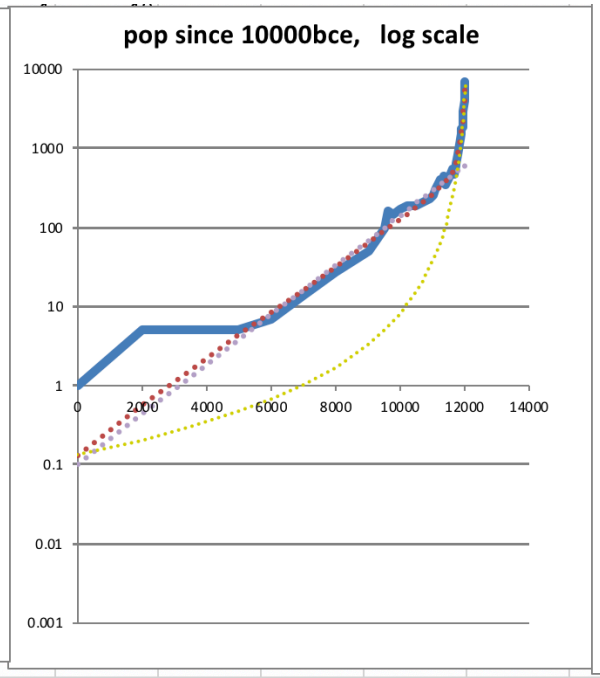
Reasons for positive feedback, *hyperbolic* growth:
How does population increase boost population increase rate?

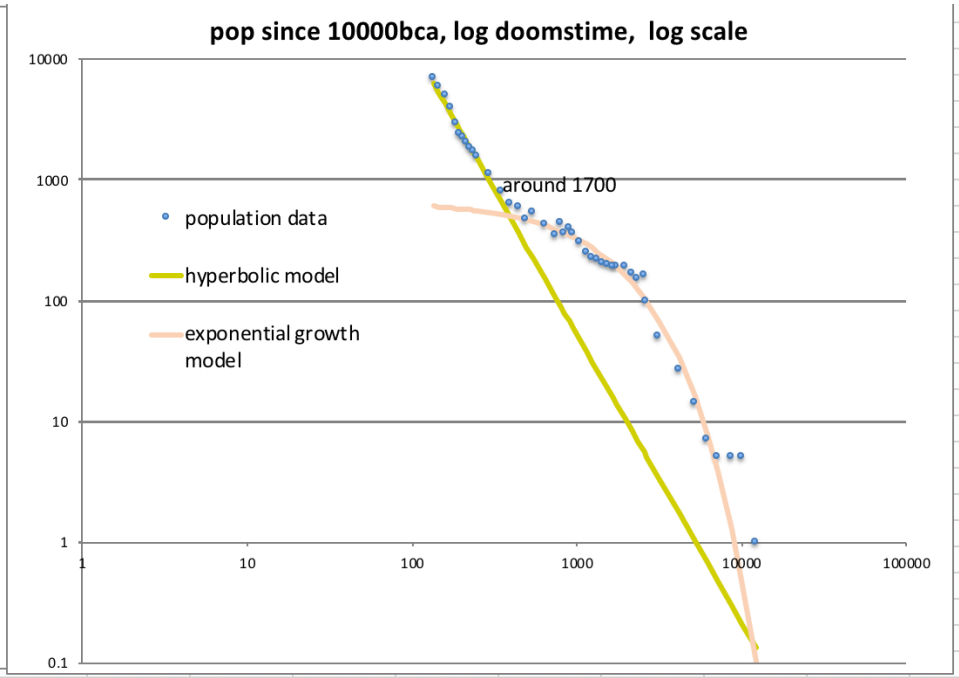
- Increased job specialization. Higher efficiency.
- Less fighting. More peace.
- Technology improves. Death rate decreases.
- Other species eliminated. Less competition.



Fitting population to hyperbolic (Doomsday) growth using Excel.





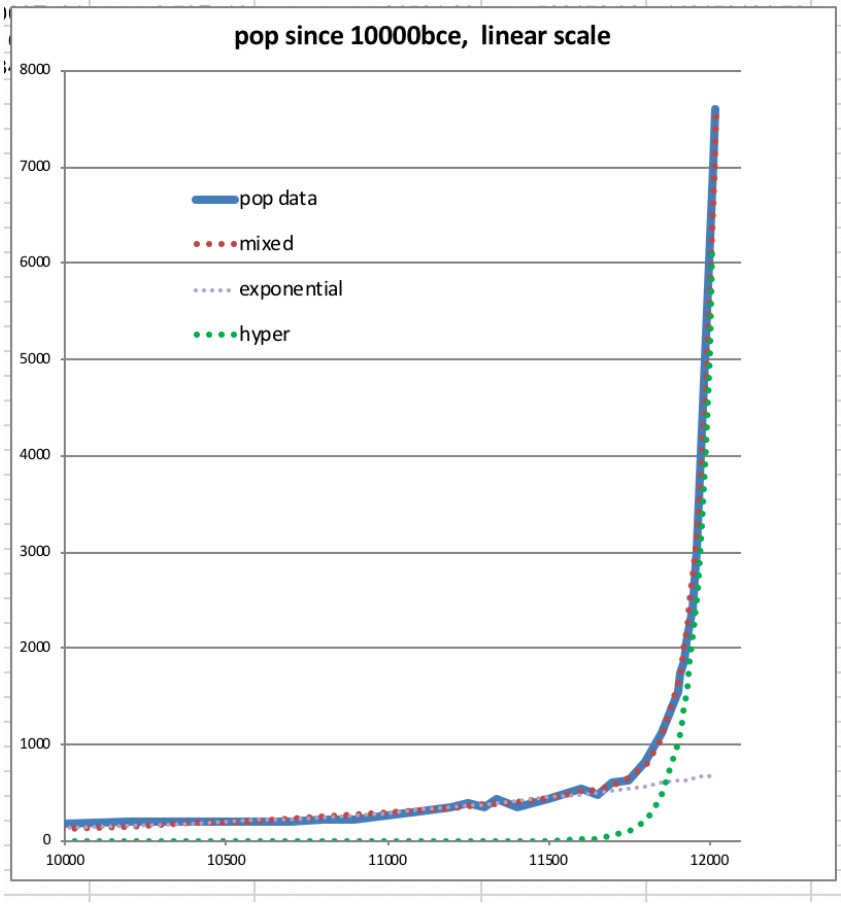


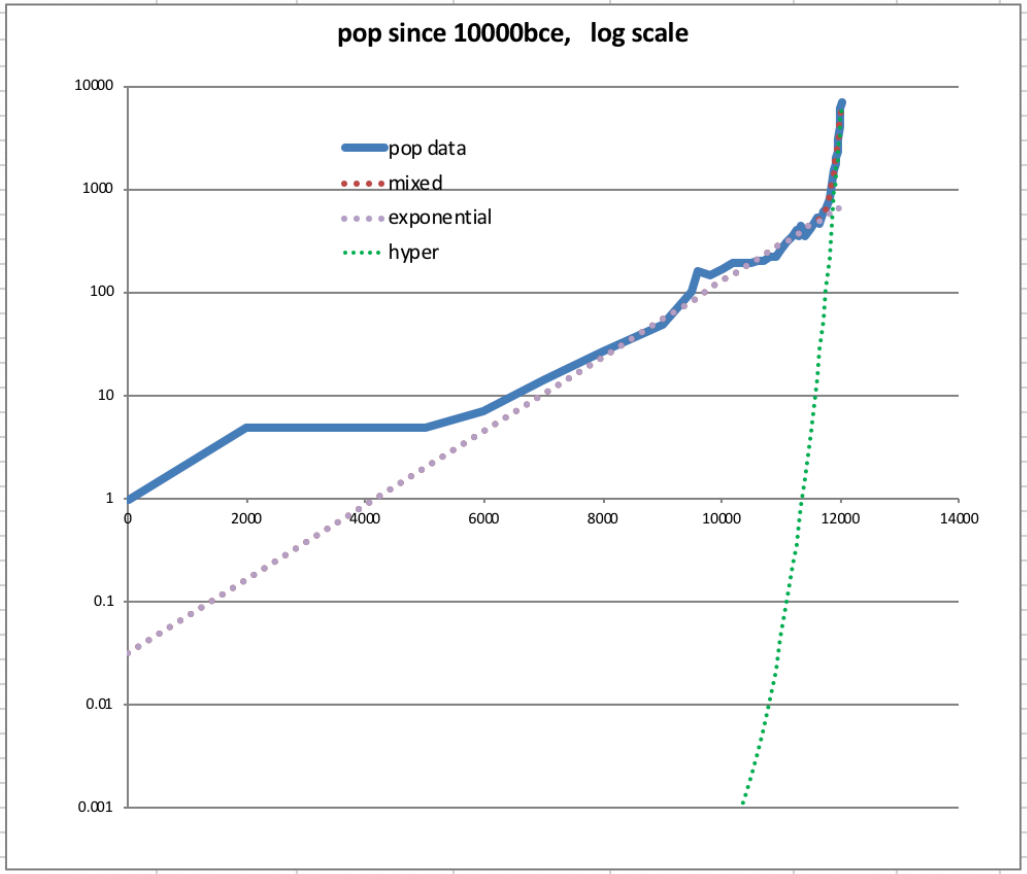
Population (10,000bce-1700) fits exponential growth.

Population (1700-present) fits hyperbolic growth.

Exponential plus hyperbolic growth fits the full range (10,000bce-2000).

Fitting population to hyperexponential growth using Excel.





Population (10,000bce-1700) fits exponential growth.

Population (1700-present) fits hyperexponential growth.

$$N_t = N_0 \exp(\alpha \exp(\beta t))$$

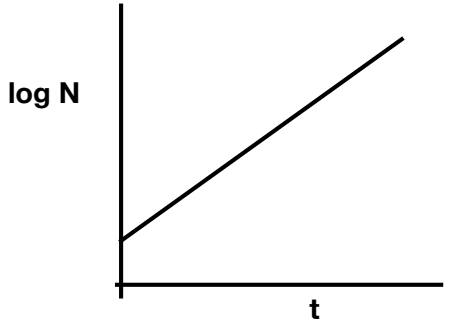
Exponential plus hyperexponential growth fits the full range (10,000bce-2000).

What is **hyperexponential** growth?

Simple exponential growth

$$\frac{dN}{dt} = \alpha N$$

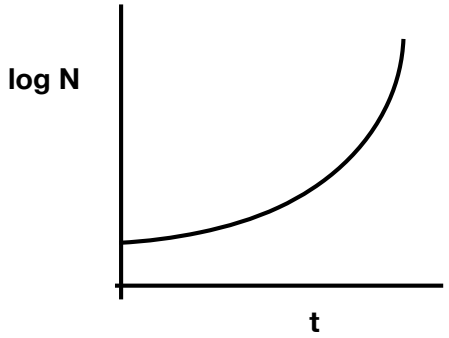
$$N_t = N_0 \exp(\alpha t)$$



Hyper exponential growth

$$\frac{dN}{dt} = \alpha \, exp(\beta N)$$

$$N_t = N_0 \exp(\alpha \exp(\beta t))$$



When the rate of growth is itself increasing exponentially, then growth is of the form exp(exp(t)), hyperexponential.
It's exponential in log space!

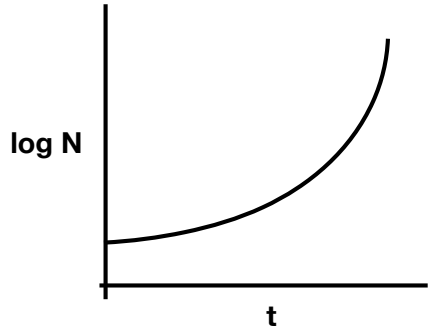
What caused **hyperexponential** growth?

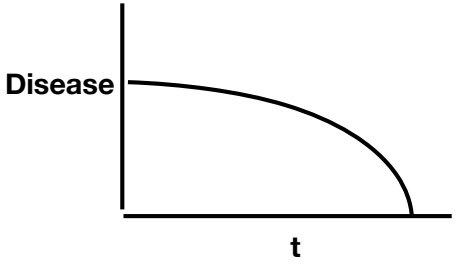
- **Growth rate is the birth rate minus the death rate. As long as they are constant, we have exponential growth.**
- **If the birth rate increases exponentially, or the death rate decreases exponentially, then growth is hyperexponential.**
- **The birth rate has not increased exponentially (but infant mortality has indeed decreased, effectively increasing the rate of successful childbearing.)**
- **The death rate has been decreasing in proportion to disease. Exponential decline of disease?**

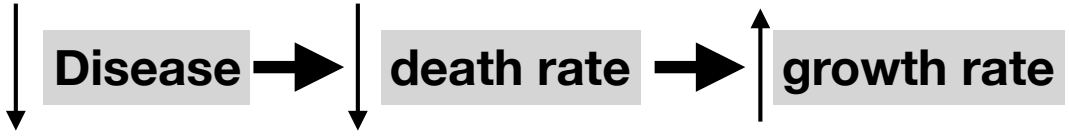
$$N_t = N_0 \exp(\alpha \exp(\beta t))$$

$$N_t = N_0 \exp(g)$$

$$\text{where, } g = \alpha \exp(\beta t))$$

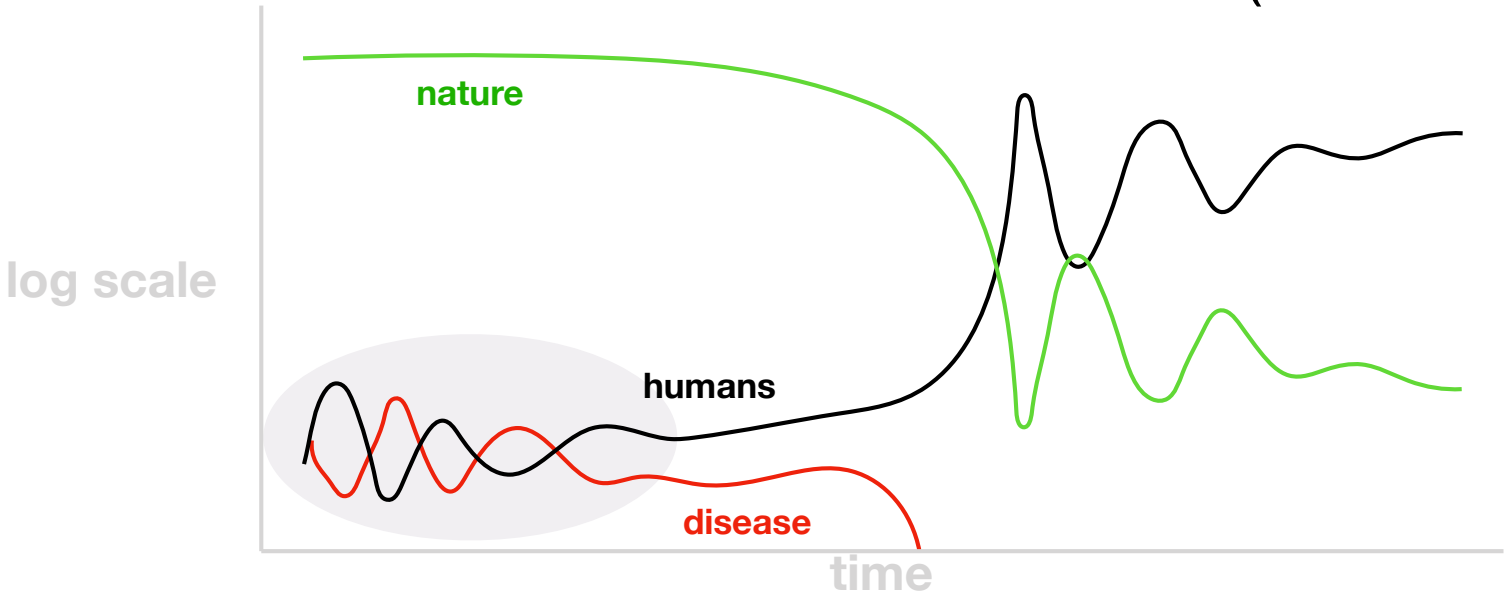




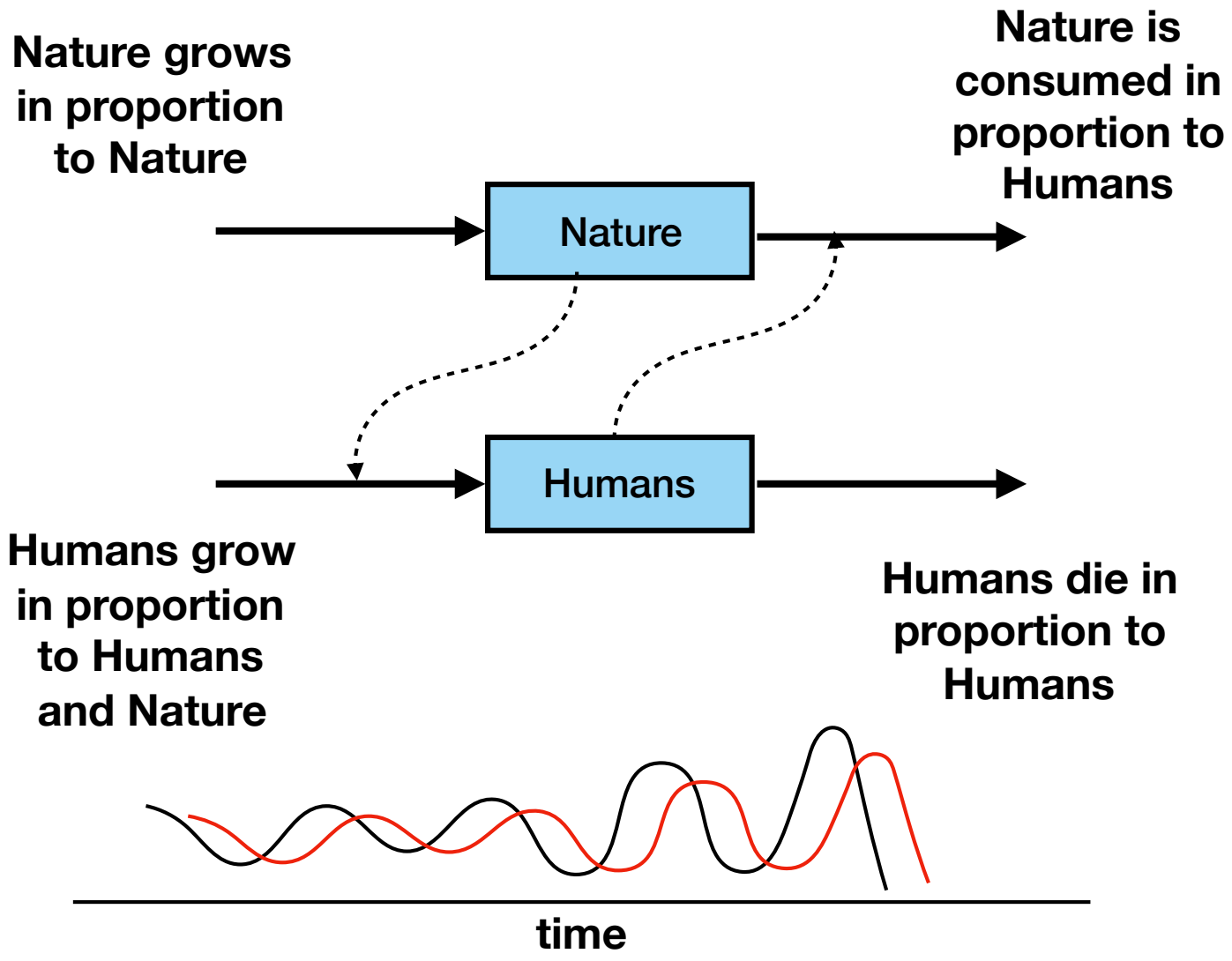


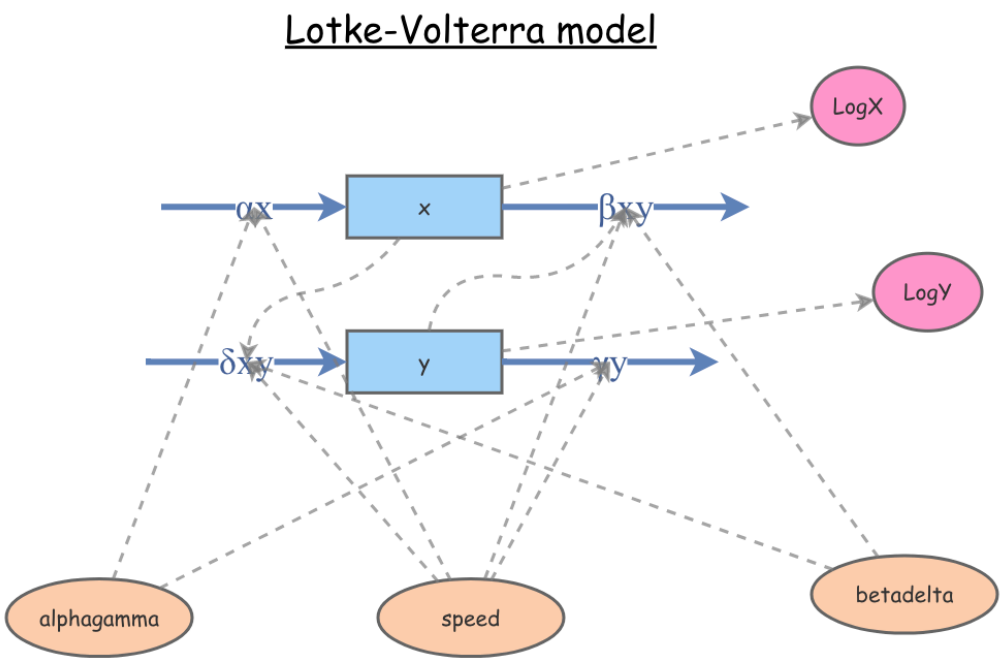
From equations to systems

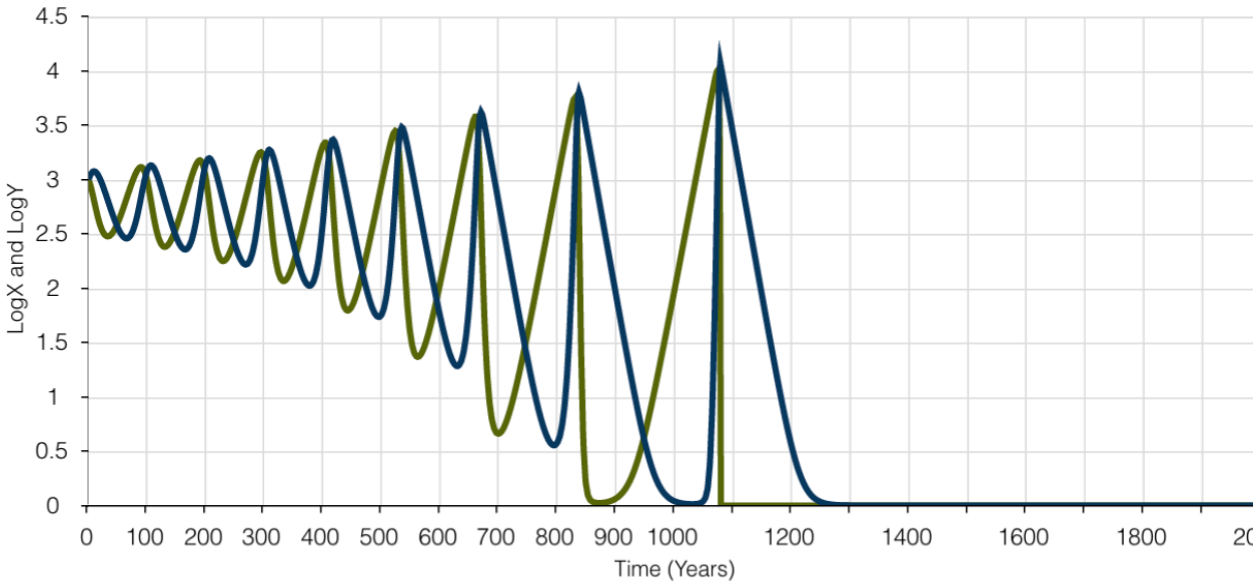
- **In ancient times**, humans were controlled by predators and disease, oscillating stably. (Lotka-Volterra model)
- In recent times, **diseases have disappeared**. While they disappeared humans experienced hyperexponential growth.
- After reaching carrying capacity, **natural systems decline** until humans decline. Then natural systems recover. Humans and Nature oscillate. (Lotka-Volterra



Lotka-Volterra model

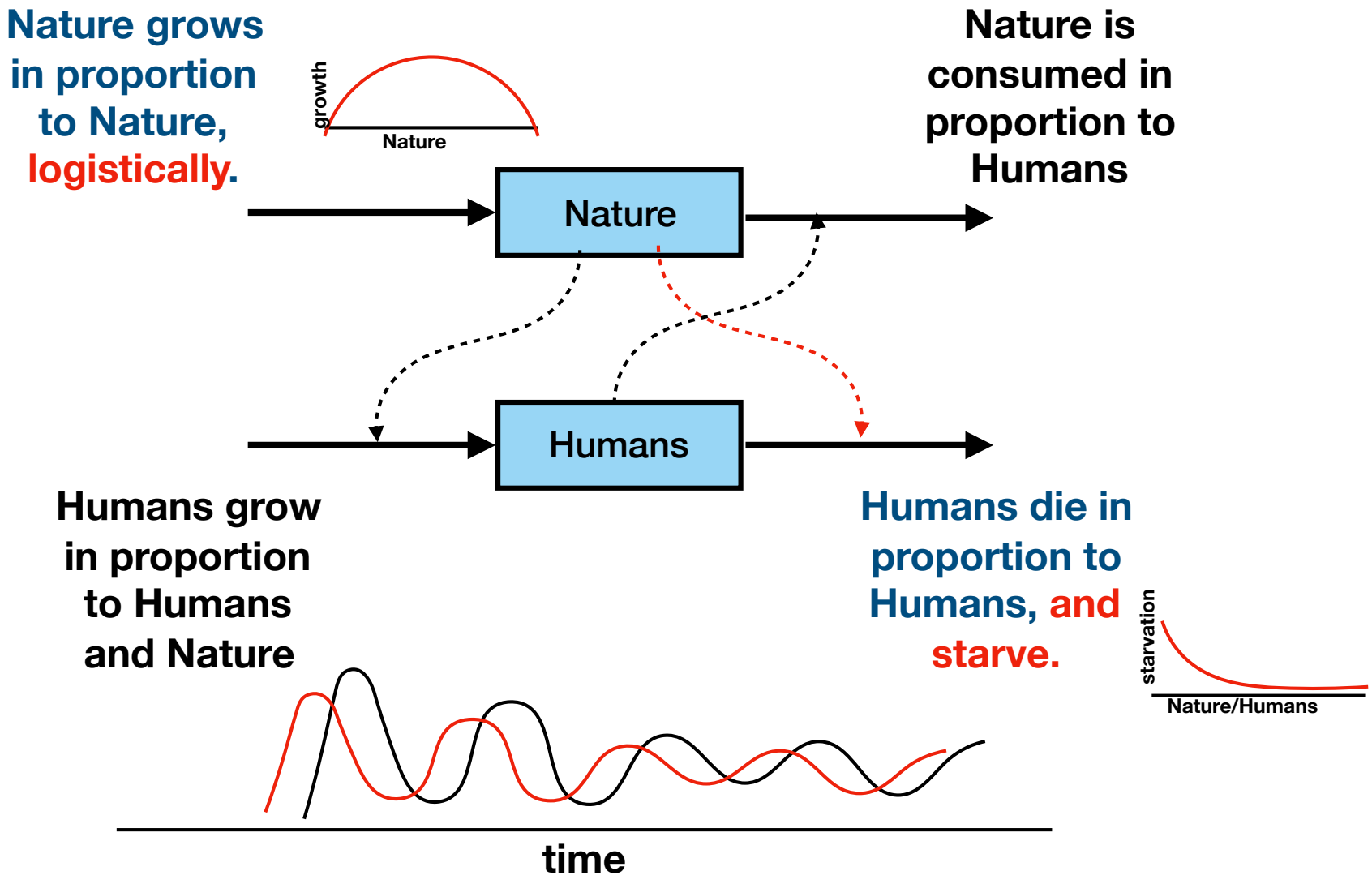




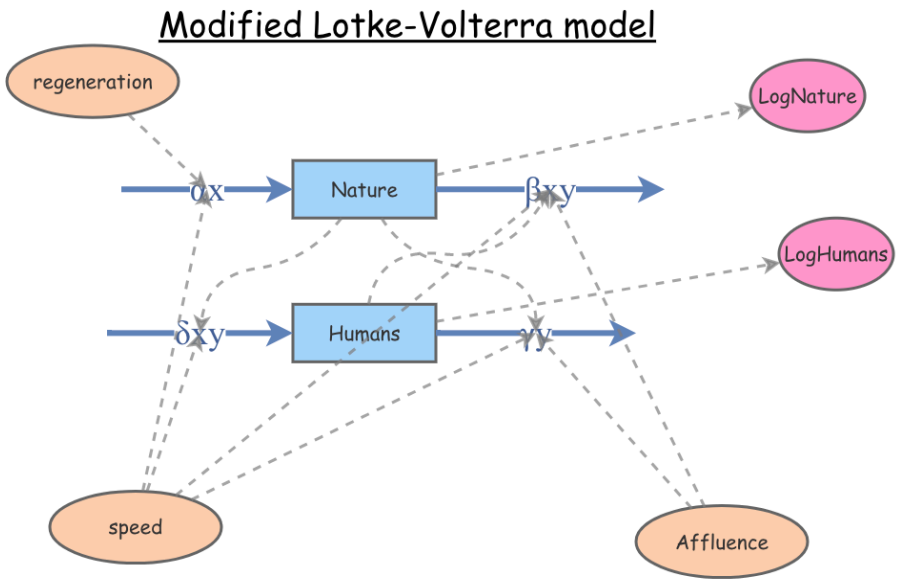


Because predator *y* death rate γy is intrinsic, *y* decreases gradually with negative feedback (negative exponential growth). But this means prey *x* populations are driven ever lower, until collapse.

Modified Lotka-Volterra model



Starvation causes damped oscillations



Model Flows

αx

Rate:

```
ceiling <- 10000
floor <- 0
regeneration <- [regeneration]
x <- (ceiling-[Nature])/(ceiling-floor)
x <- x*regeneration*[Nature]*[speed]
return x
```

βxy

Rate:

```
PAT <- [Affluence]*[Humans]
y <- PAT*[Nature]*[speed]
return y
```

γy

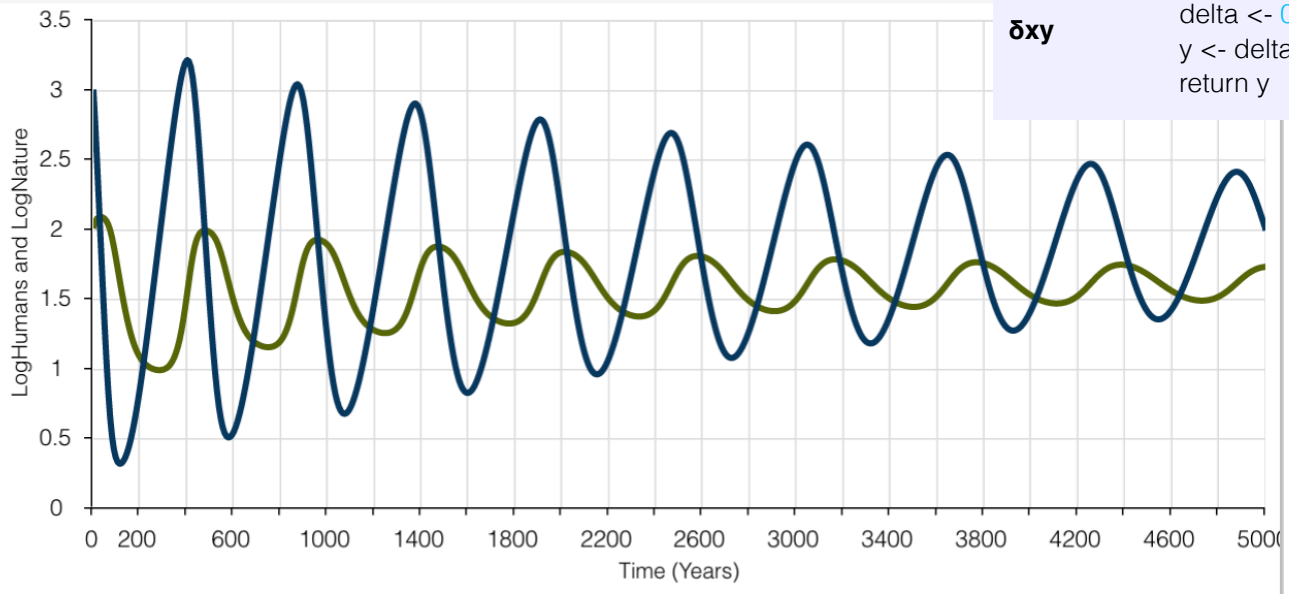
Rate:

```
gamma <- 0.005   ## natural death rate
foodpp <- ([Nature]+1)/([Affluence]*[Humans] +1)
unfed <- (1.0-gamma)/(1+5*foodpp)   ## fraction unfed
gy <- (gamma+unfed)*[Humans]*[speed]
return gy
```

δxy

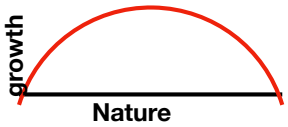
Rate:

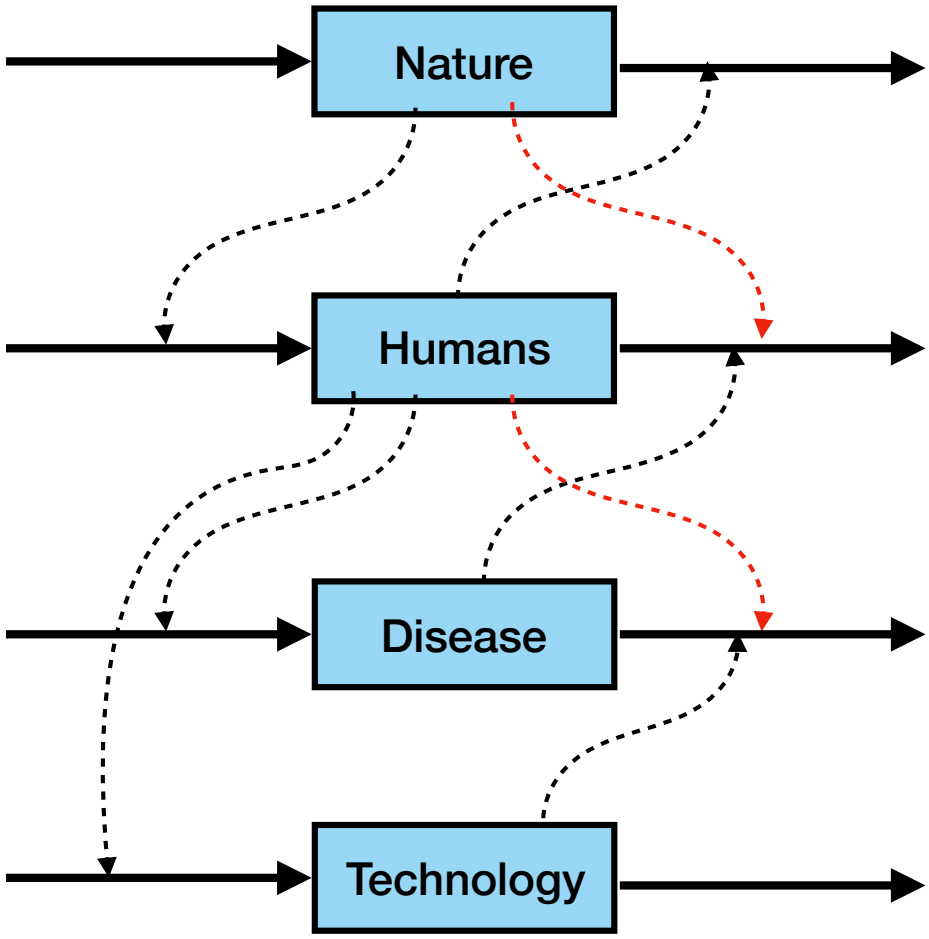
```
delta <- 0.00009
y <- delta*[Humans]*[Nature]*[speed]
return y
```

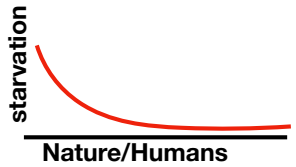


Now predator *Humans* death rate γy is extrinsic, depending on food availability (*Nature*), *Humans* decreases more abruptly when food is scarce, saving *Nature* from collapse. **Damped oscillation**.

Tandem Modified Lotka-Volterra model







Humans die in proportion to Disease, and still starve when Nature is low.

Disease grows in proportion to Humans and Disease

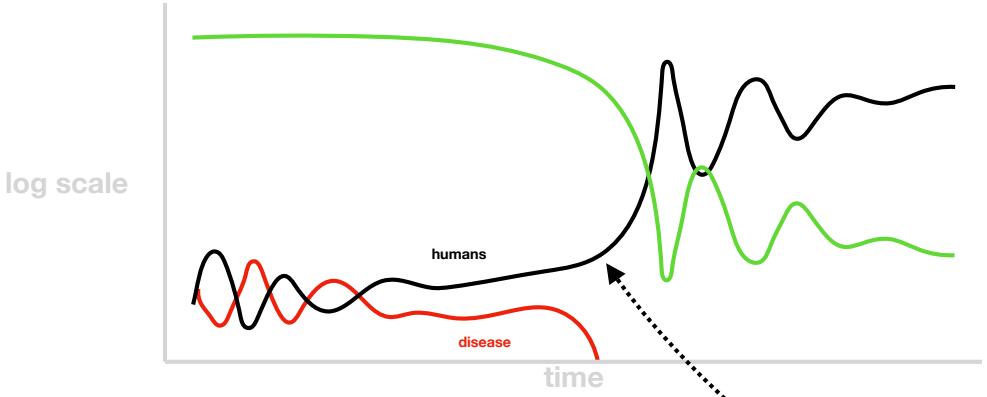
Diseases are eradicated in proportion to Humans, and Technology.

Learning is proportional to Humans?

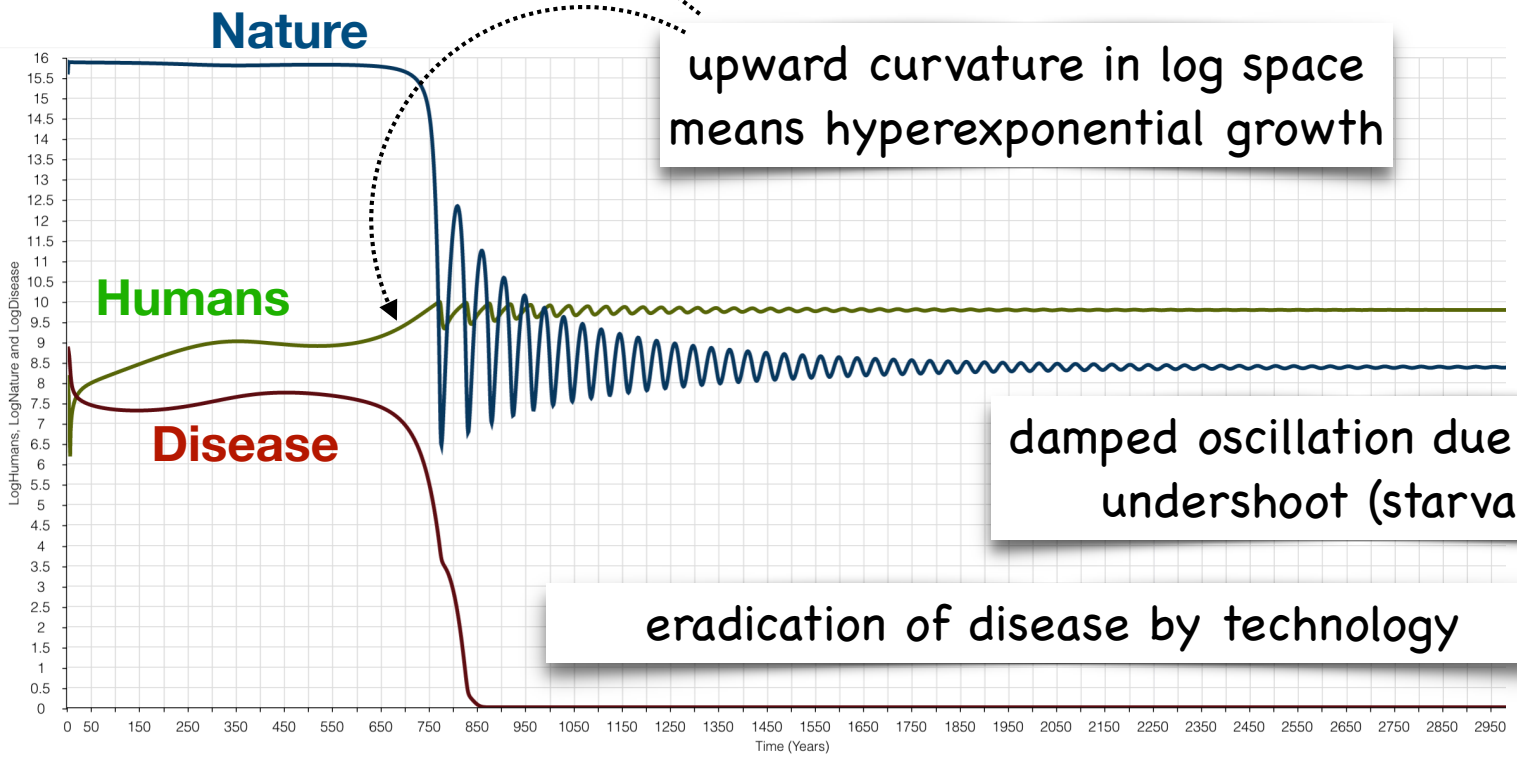
Forgetting is proportional to Technology?

Tandem Modified Lotka-Volterra model

with Technology



log scale

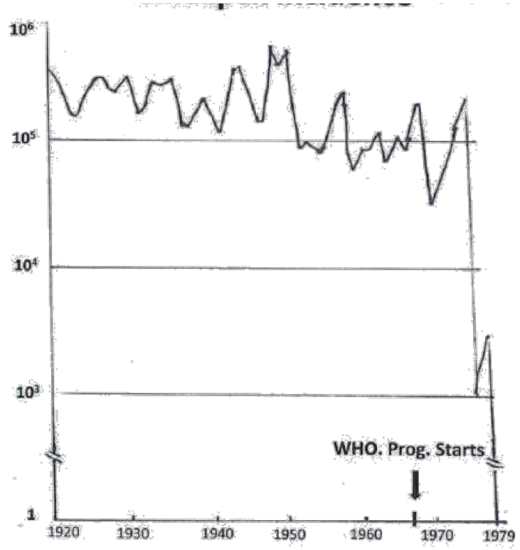


upward curvature in log space means hyperexponential growth

damped oscillation due to over/ undershoot (starvation)

eradication of disease by technology

The amazing end of diseases in the 19-20 century



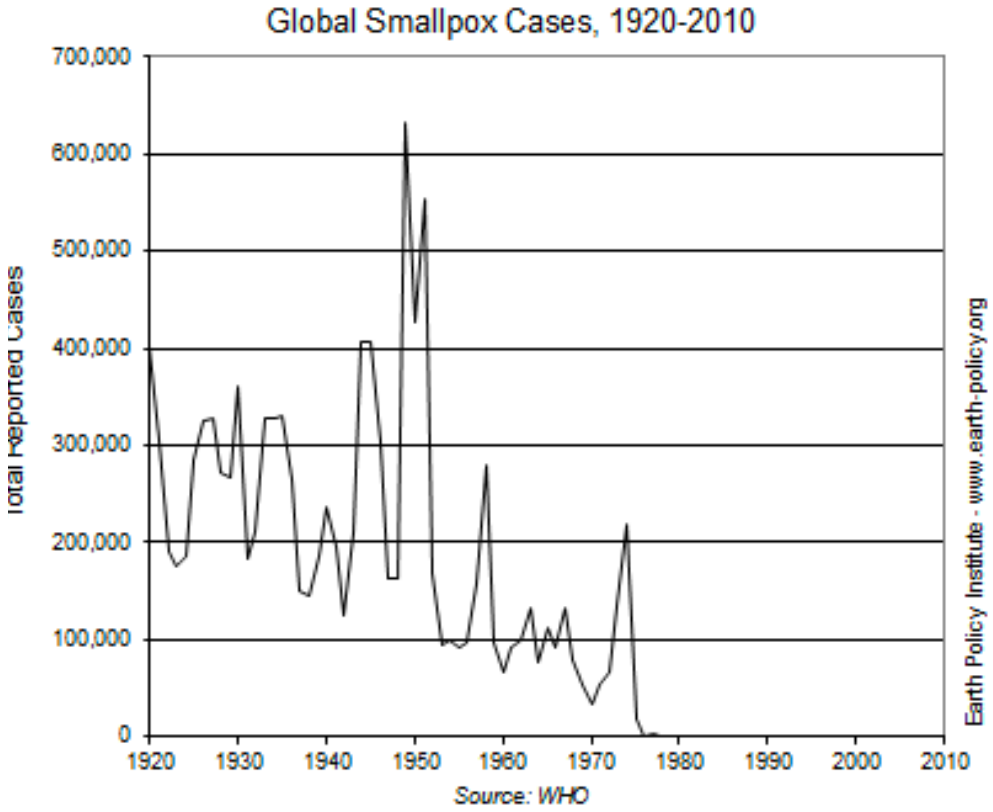
Source-WHO Final Report of the Global Commission for the Certification of Smallpox Eradication, 1980

Note:

Global population:
1920 - 1,900 million,
1950 - 2,500 million,

You may wish to estimate how Smallpox spread widely in olden time, Considering the reporting nation could be very low.

Figure 2: Smallpox incidence worldwide from 1920 to 1980.



Arita I (2013) Smallpox Eradication and Human Morality in Evolution. J Vaccines Vaccin 4: 188

The multipurpose nature of Technology

- How does technology affect other components of the tandem modified Lotka-Volterra model?
- Biocapacity?
- Birth rate?
- Extinction of competing species?

Debate 4 topics?

Friday 3/30

my list:

- Disease: good or bad?
- Are we smarter than yeast?
- Technology is unique to humans.
- Is global genocide in our future?

Next time: moral predicaments

- Read Hardin. (it is not impenetrable)