Human Population 2018

Lecture 3
Exponential Growth
Systems dynamics
Human population history

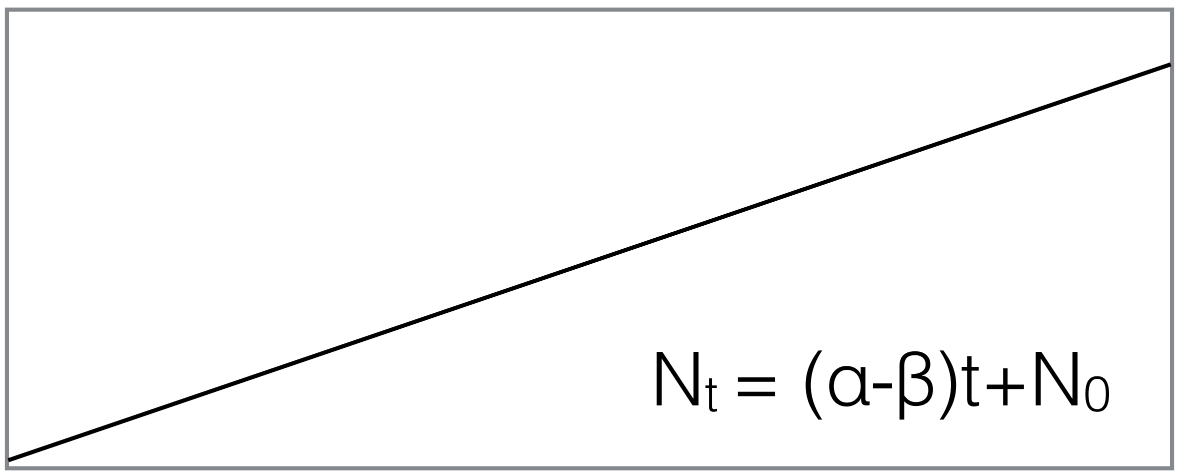
Questions from the reading?

pp 16-36
exponential growth
birth rate, death rate
poverty

Model 1. Linear growth

$$\frac{dN}{dt} = \alpha - \beta$$

births deaths

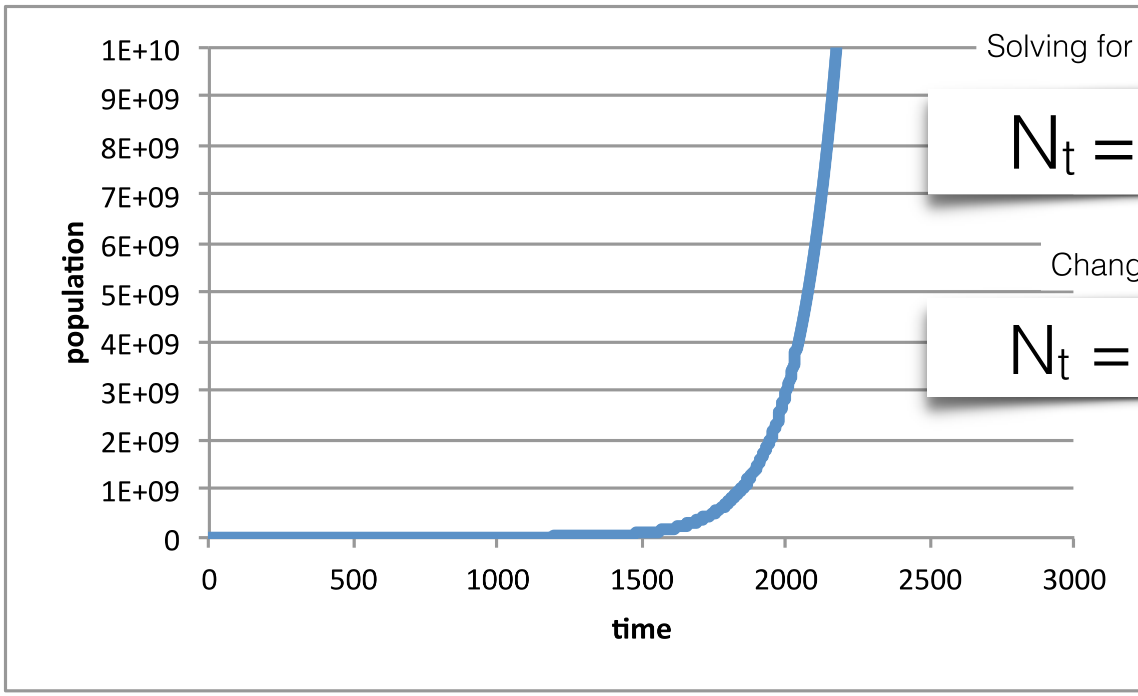


Linear growth. Constant increase or decrease.
Makes sense for lots of systems, but <u>*not*</u> *for population growth* since a constant number of births requires a *ever-decreasing birth rate.*

Model 2. Exponential growth

$$\frac{dN}{dt} = \alpha N - \beta N = (\alpha - \beta)N$$

births deaths



Solving for $\mathbf{N_t}$ puts **t** in the exponent

$$N_t = (1+\alpha-\beta)^t N_0$$

Changing to base **e** gives,

$$N_t = N_0 \exp((\alpha-\beta)\cdot t)$$

Population growth is proportional to population. This is a <u>natural</u> population growth curve for the case of :

no predation,
no shortages,
no social unrest,
no limits of any kind.

"A lack of appreciation for what exponential increase really means leads society to be disastrously sluggish in acting on critical issues"

Dr. Thomas Lovejoy -- Smithsonian Institution

Persian legend

Reward for making a beautiful chessboard

Put 2^n rice grains on n=64 squares.

Total number of rice grains received after filling in all 64 squares:
18,446,744,073,709,551,615

Number of rice grains in a ton of rice: **64,000,000**

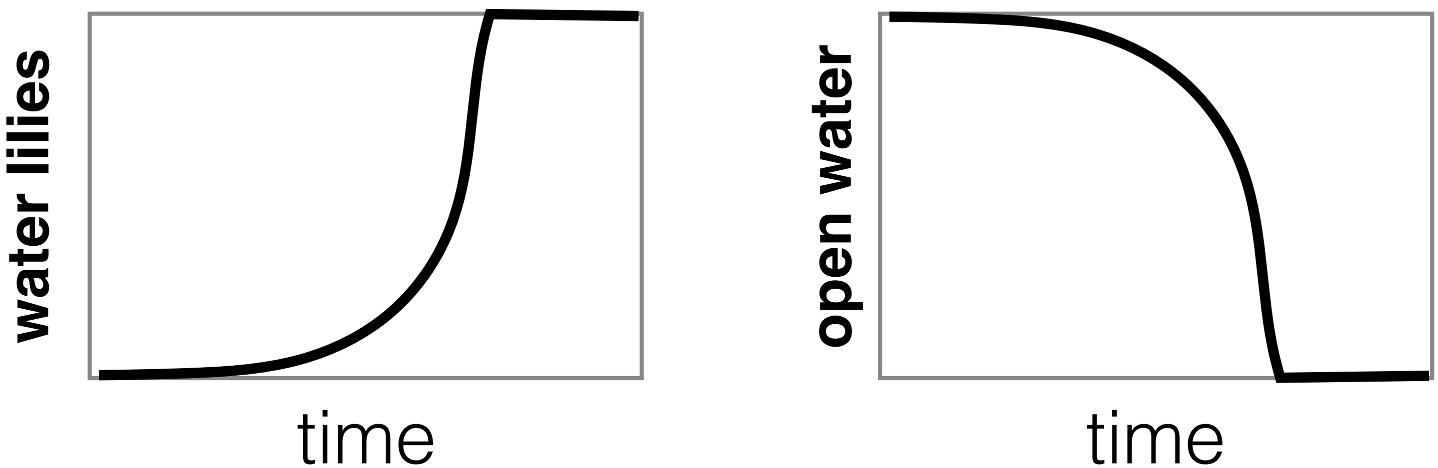
Tons of rice awarded: **288,218,750,000**

Size of cooking pot large enough to hold this much rice: **5 mi wide, 5 mi tall.**

French riddle

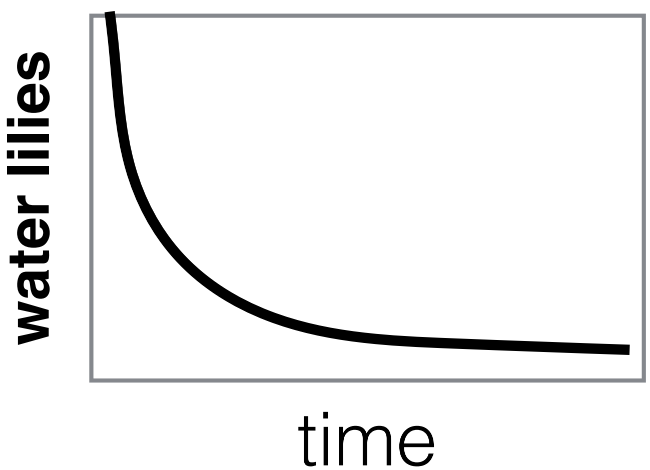
A water lily plant doubles in size every day, covering the pond in 30 days. When was the pond half-filled with water lillies?

<u>exponential decline</u> of pond open water



Forest cover remaining during a forest fire, fossil fuels remaining as car usage grows exponentially, bank reserves during a run on the banks, all follow (may follow!) <u>exponential decline</u>.

...Not to be confused with negative exponential growth....

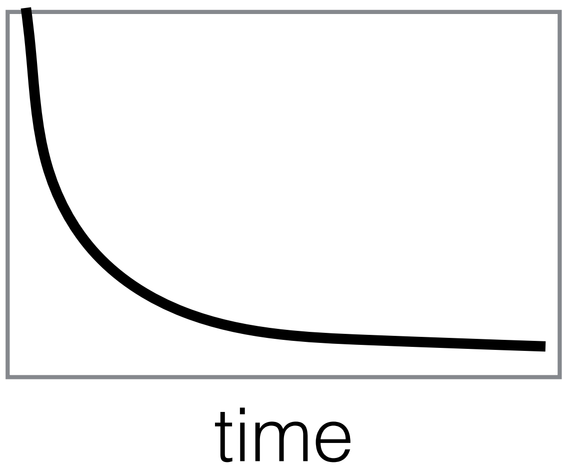


water lilies in the fall.

Often when we say exponential decline, we are talking about negative exponential growth.

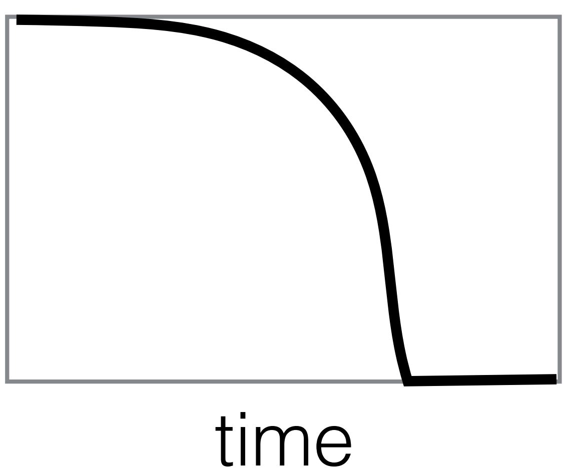
zero is the intrinsic, negative limit.

Negative exponential growth



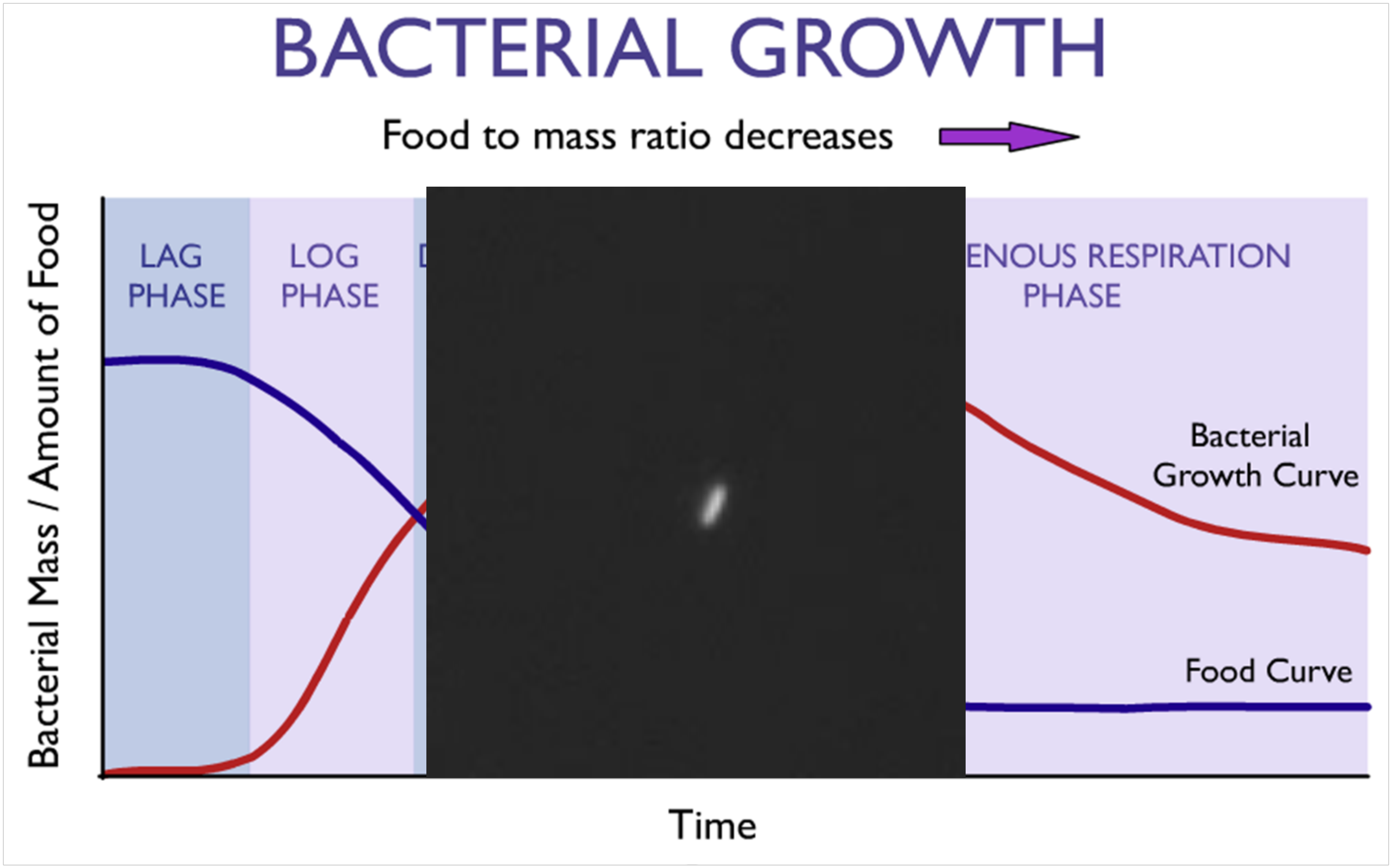
Negative feedback.
Slope of decline constantly decreasing.
Follows exponential equation. Slope is a function of y.

Exponential decline



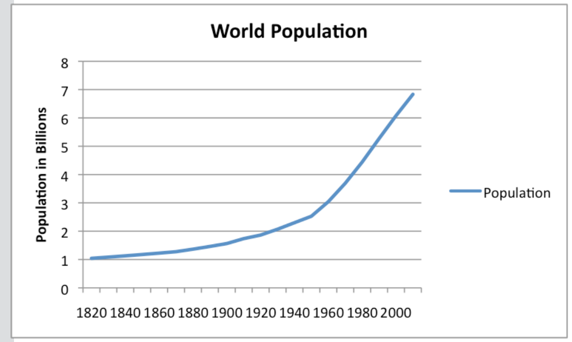
Not an exponential function.
Slope is not a function of y.
Externally caused (function of something else, that is presumably growing.)
Slope of decline constantly increasing to zero.

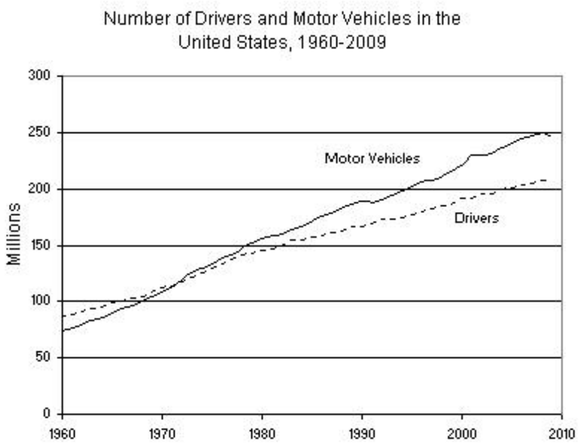
Things that grow exponentially...

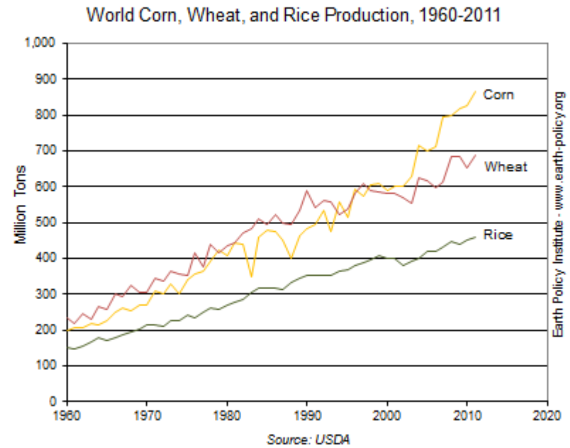


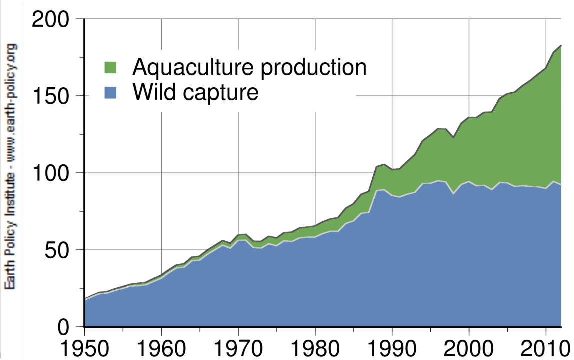
Rate of growth depends on number of bacteria

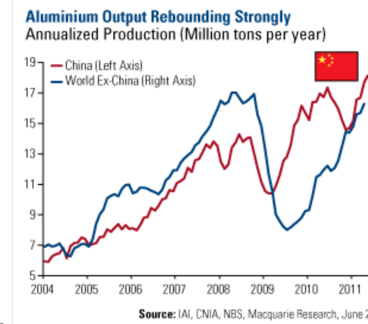
Things that are growing

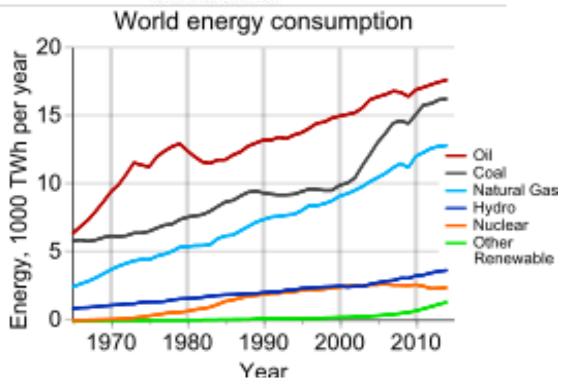


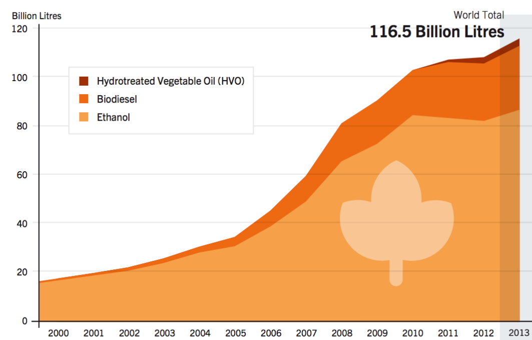


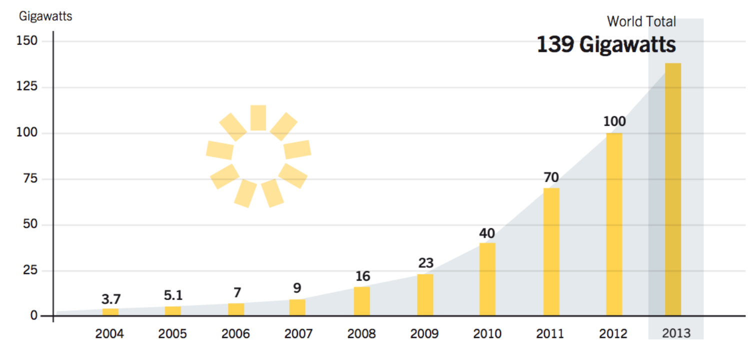


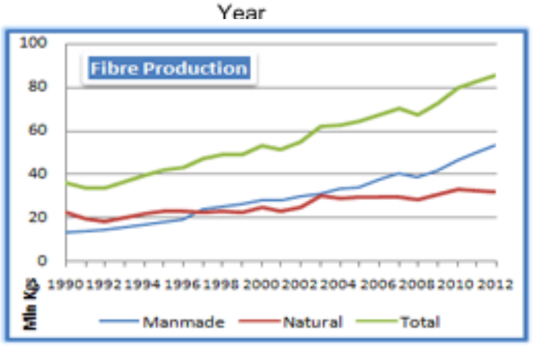


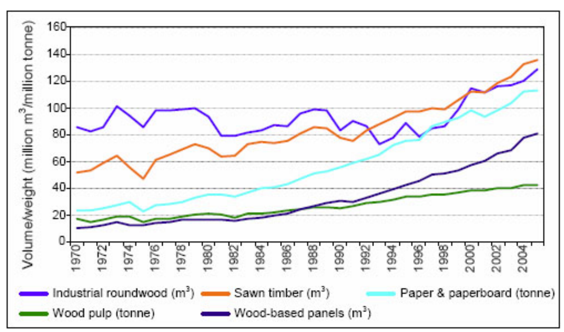


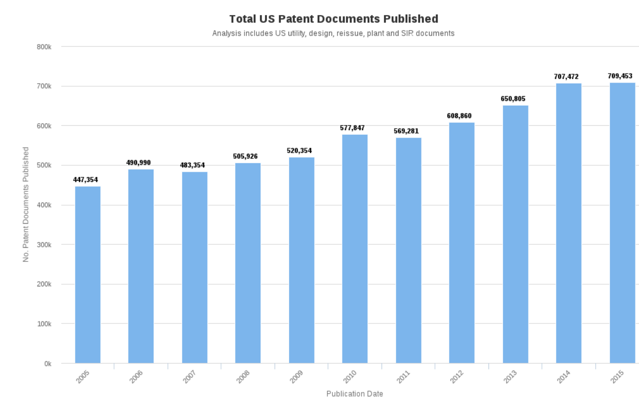






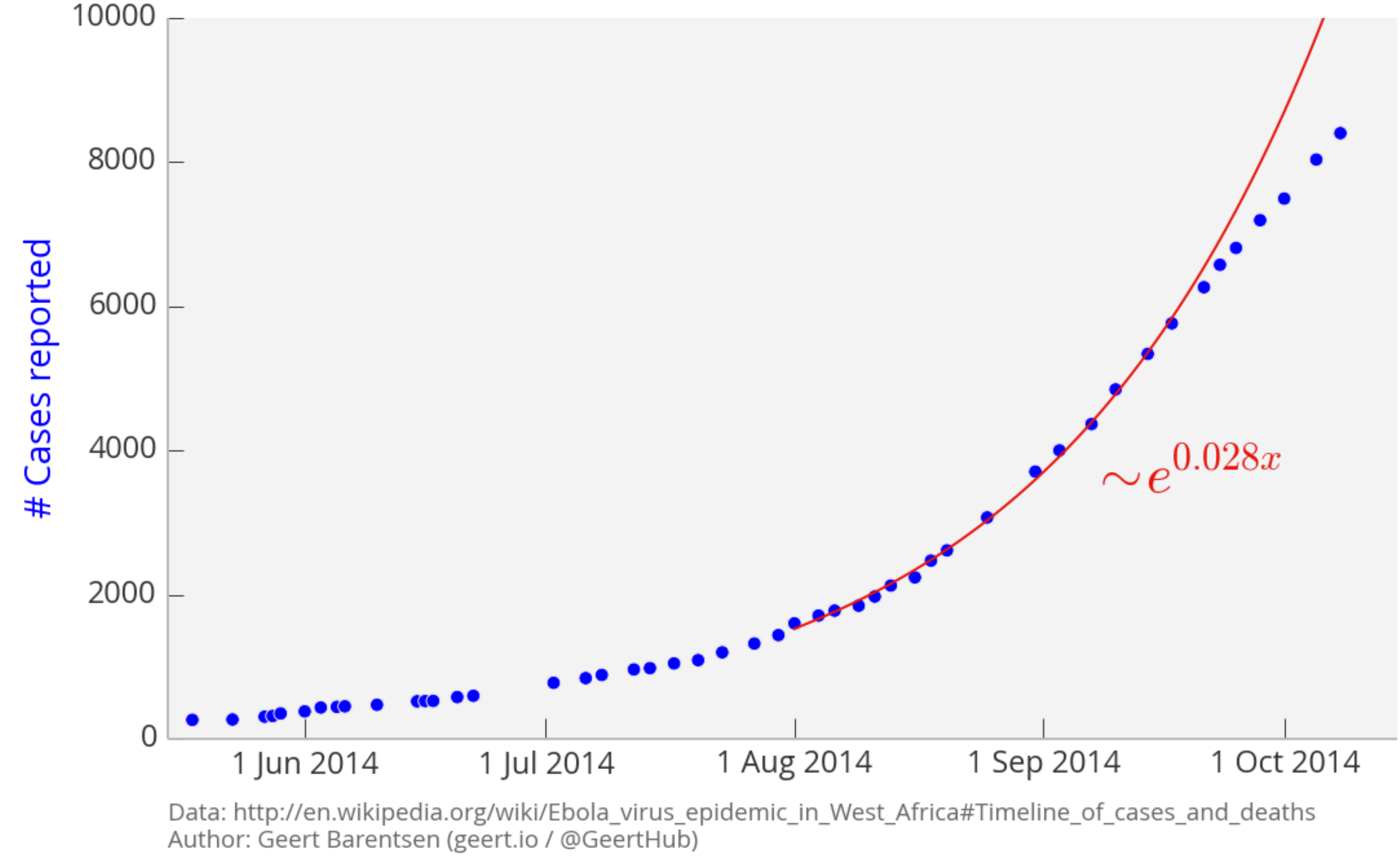






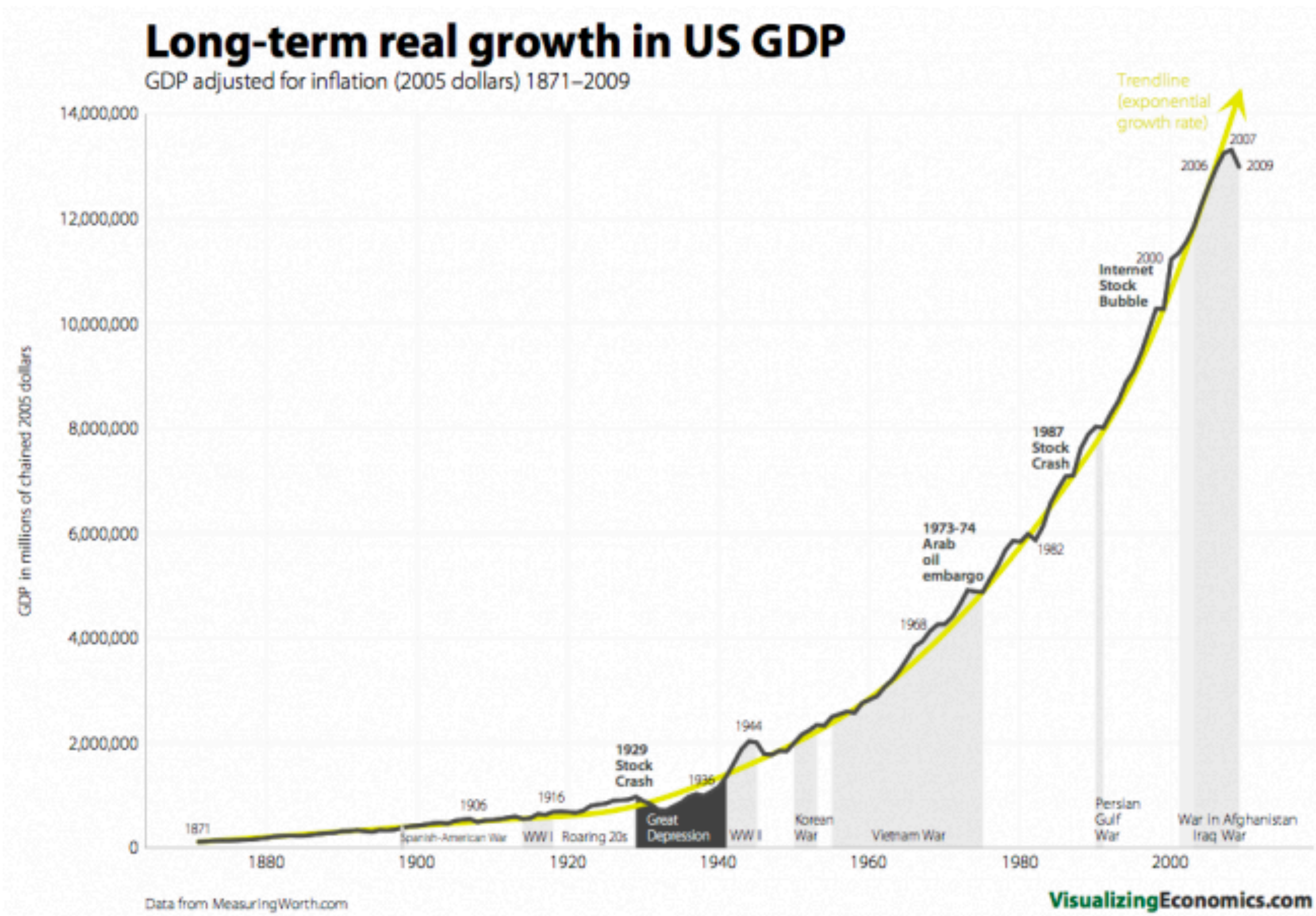
Things that grow exponentially...

Spread of ebola ...at least for a short while...



New infection rate is a function of number infected.

The economy



The rate of growth in investments depends on the volume of investments.

How do you know it's exponential? Plot it in Log scale

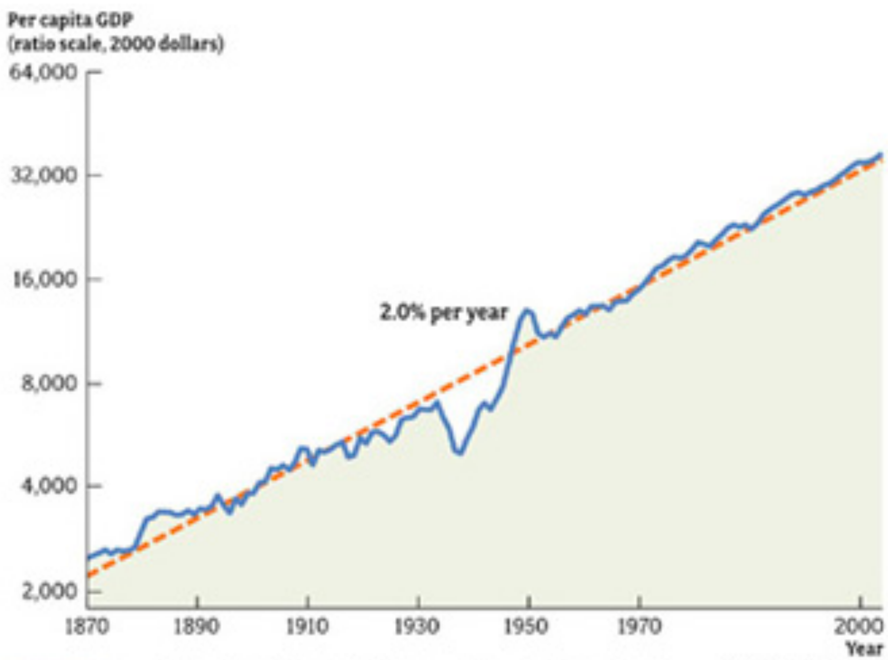
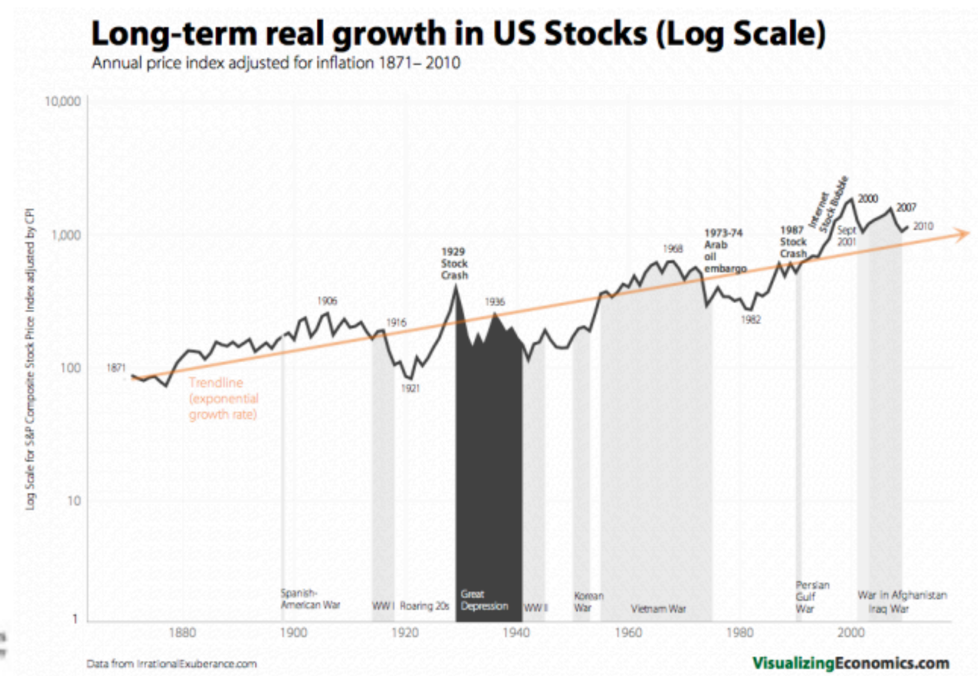


FIGURE 3.5 Per Capita GDP in the United States, 1870–2004: Ratio Scale

Macroeconomics, Charles I. Jones



computationallegalstudies.com/2010/10/29/exponential-growth-rate-of-us-stocks-since-1871-via-visualizing-economics

the math of exponential growth

- When the rate of growth is proportional to the current amount, then growth is exponential. It increases by the same factor at each time period.

- Examples of constant exponential growth:
  1,2,4,8,16,32,64 , etc
  1.01, 1.0201, 1.030301, etc

- $\mathbf{N_t = N_0 g^t}$, where *t* is number of growth cycles, N_0 is the starting amount, *g* is the "*growth factor*" per growth cycle.

example: population growth in the USA in recent years

- Growth factor for humans in the US is measured by Total Fertility Rate (TFR) = number of children per woman = 2.5. Since women are 1/2 the population, growth rate **g** = 2.5/2 = 1.25 over one cycle.
- Cycle time for humans is approximately the average age at time of birth. Let's say it is 25. **t** is in units of 25 years.
- Given the current US population, N_0=3e8, what will the population be in 2068?

$\mathbf{N_t = N_0 g^t}$ = $3e8(1.25)^{((2068-2018)/25)} = 3e8(1.25)^2 =$ 468,750,000

the math of exponential growth

- Converting to base-*e*...
- $N_t = N_0 g^t$
- Define *growth rate*: $r = \ln(g)$
- Then, $r = \ln(N_t/N_0) / t$

  The *growth rate* is the natural log of the average *growth factor*.
- Solving for N_t,

  $N_t = N_0 \exp(r \cdot t)$

	growth factor *g*	growth rate *r*
	2	1.69
	1.5	0.41
	1.1	0.095
	1.01	0.0099
	1.001	0.001
if r << 1,	1 + r	r

Simplifying approximation

For slow growth, the rate equals the growth factor minus one.

example: population growth in the USA in recent years (*simplified*)

- Growth factor based on TFR = 1.25 over 25 years.
- **Linear approximation** gives (1.25-1.00)/25 = 0.01/year
- Current US population is N_0=3.0e8. What will the population be in 2068?
  $N_t = N_0 \exp(r \cdot t)$ = 3e8 exp(0.01*50) = 494,616,381

Off by 25,866,381 !
Verdict: the simplified method is only good enough for coffee table discussions.
For serious work use $N_t = N_0 g^t$

doubling time

- T_d = doubling time = time when $N_t/N_0 = 2$
- $r = \ln(N_t/N_0) / t$
- T_d = **ln(2)/r**
- $\ln(2) = 0.693 \approx 70\%$

$T_d \approx \frac{70\%}{r}$ ...where r is in % units.

doubling time quiz

rate	doubling time
10%/year	
1%/year	
2%/day	
0.5%/day	

negative growth: half-life

- $T_{1/2}$ = half-life = time when $N_t/N_0 = 1/2$
- $r = \ln(N_t/N_0) / t$
- $T_{1/2} = \ln(1/2)/r =$ **$-\ln(2)/r$**
- $-\ln(2) = 0.693 \approx 70\%$

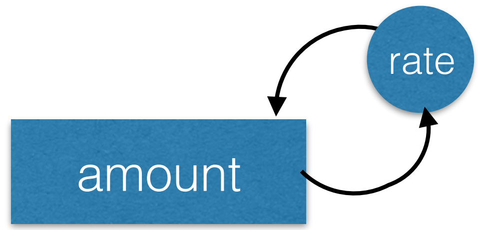
$$T_{1/2} \approx \frac{70\%}{r}$$

...where r is % decrease.

half-life quiz

rate	half-life
-10%/year	
-1%/year	
-2%/day	
-0.5%/day	

Types of feedback cycle



Rate of change of amount depends on amount.

finite element eq. for feedback

- $N_{t+1} = N_t + f(N_t)$

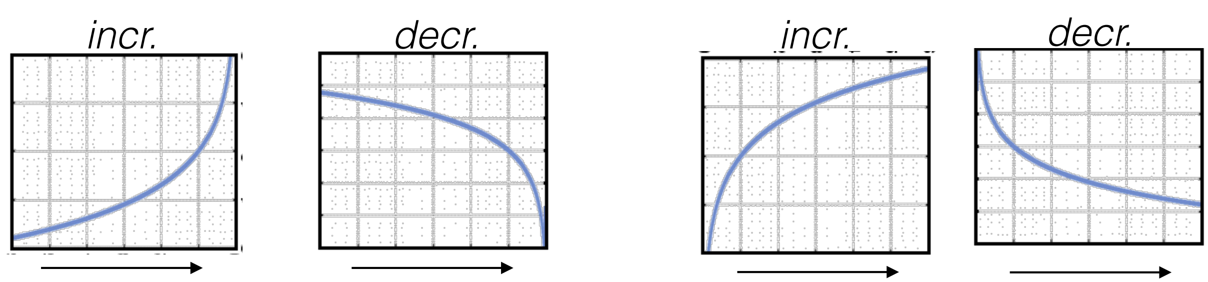
- Examples: $N_{t+1} = N_t + \alpha N_t$ *positive feedback*

  $N_{t+1} = N_t - 1/2\ N_t$ *negative feedback*

  $N_{t+1} = N_t + e^{N_t}$ *positive feedback*

  $N_{t+1} = N_t + 1/N_t$ *negative feedback*

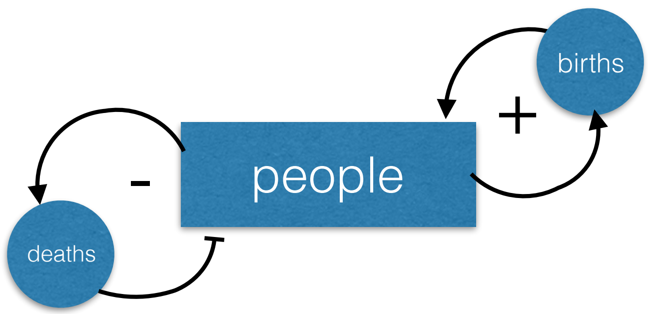
types of feedback



positive *negative*

exponential

Positive exponential growth has positive feedback.
Negative exponential growth has negative feedback.



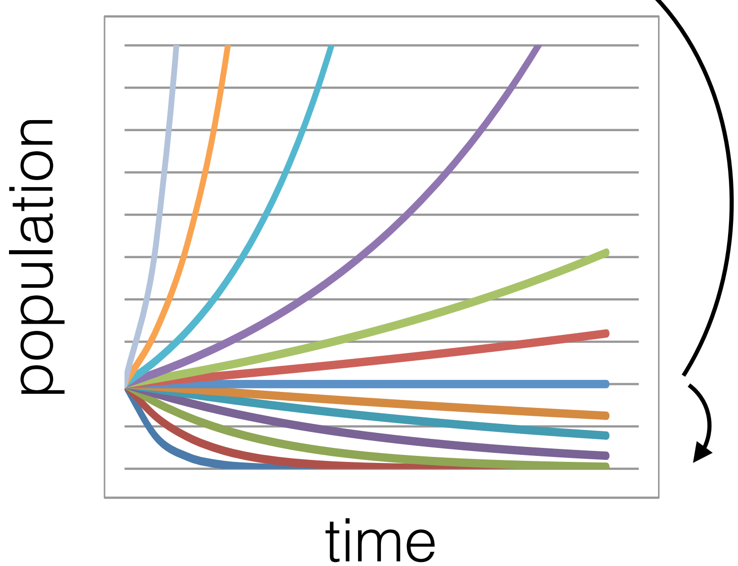
- $N_{t+1} = N_t + \alpha N_t - \beta N_t$

$N_t = N_0 \exp(r \cdot t)$

- $N_t = N_0 \exp((\alpha - \beta) \cdot t)$

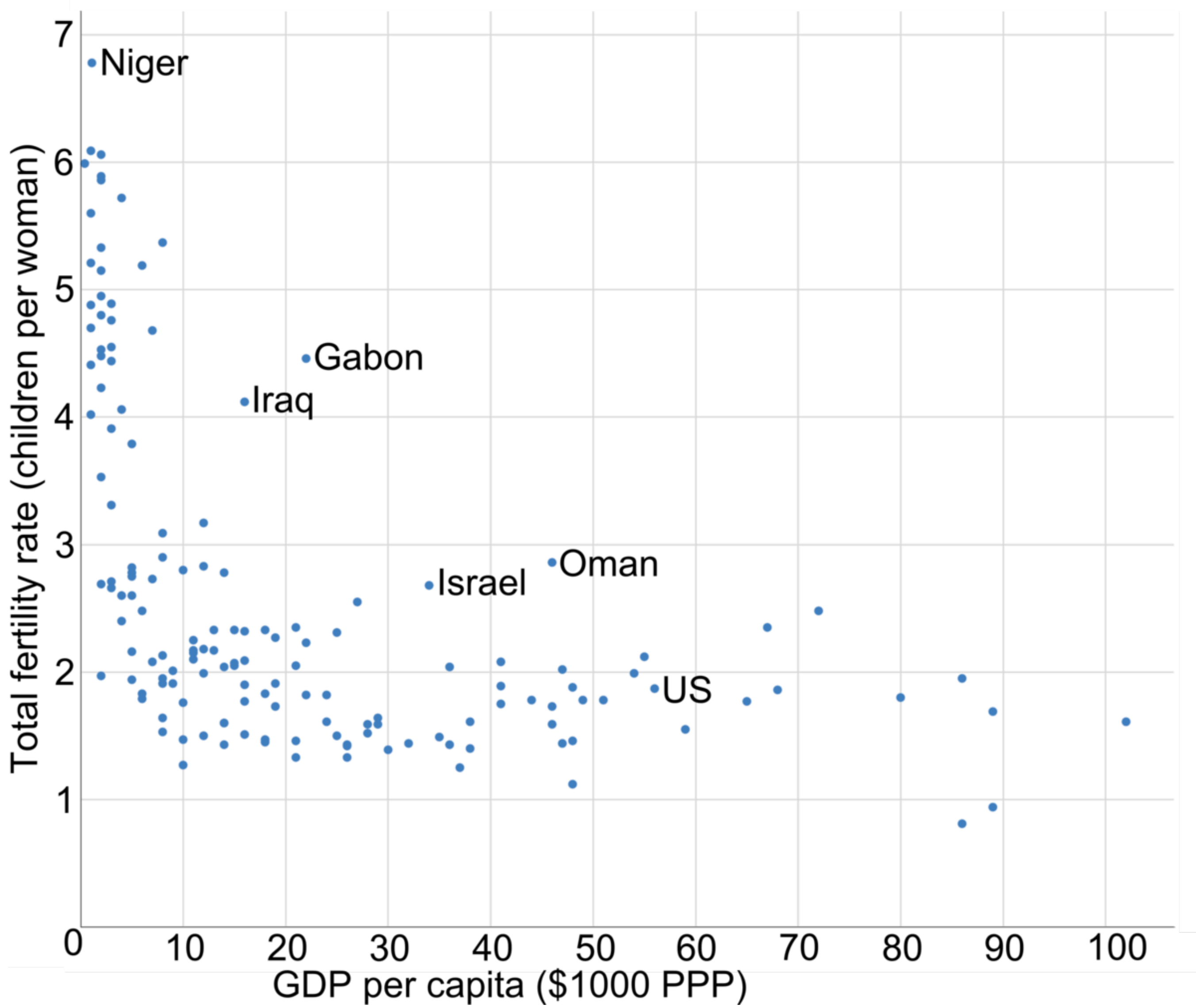
where α is the birth rate
and β is the death rate

birth rate - death rate
increasingly **positive**



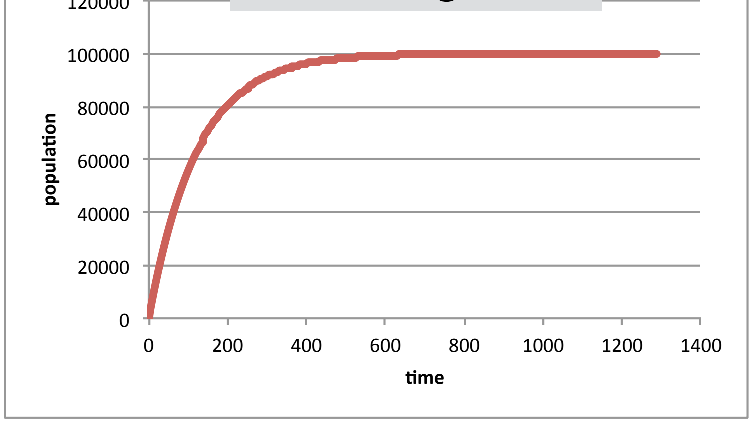
birth rate - death rate
increasingly **negative**

Poverty and population growth cycle



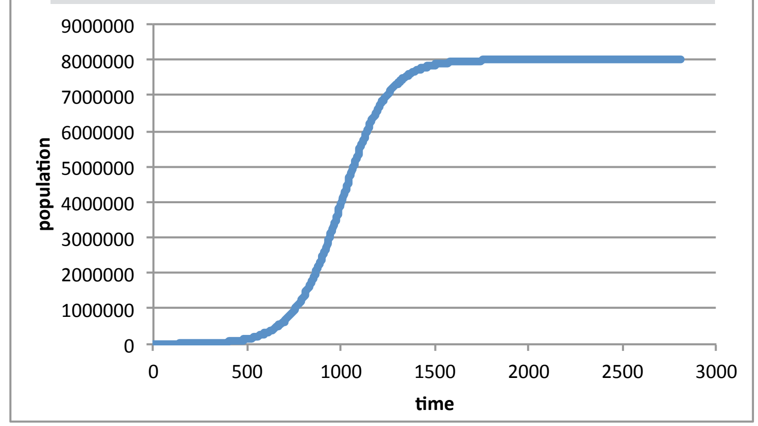
Growth with negative feedback

Limited growth



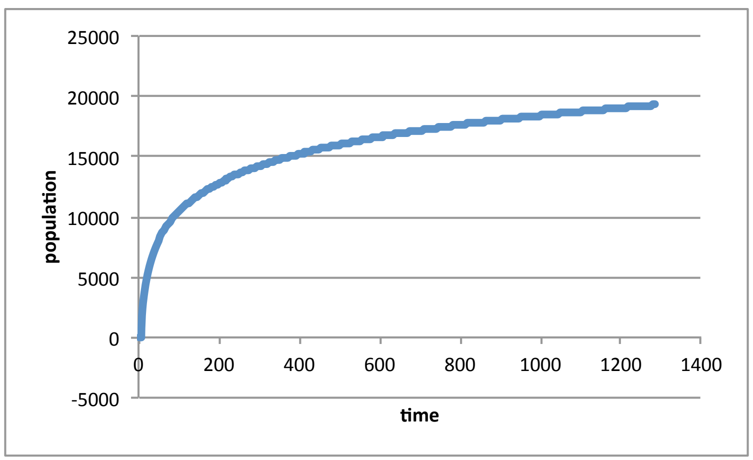
$$N_{t+1} = N_t + \alpha(N_{max}-N_t)$$

Limited exponential growth



$$N_{t+1} = N_t + \alpha\, N_t(N_{max}-N_t)$$

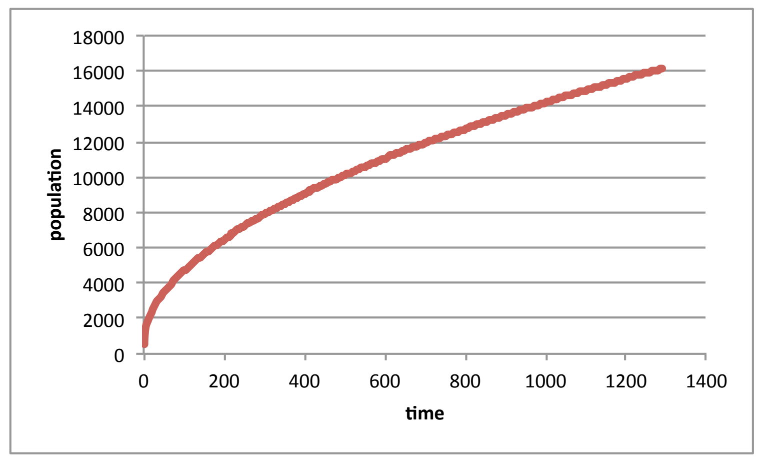
Unlimited, always slowing



$$N_{t+1} = N_t + 1/t$$

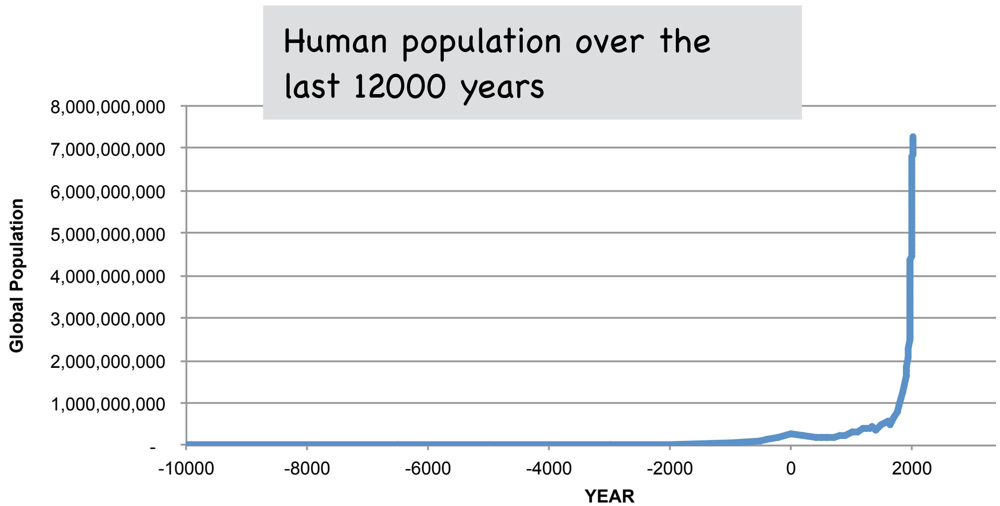
$N_t = \log(t)$

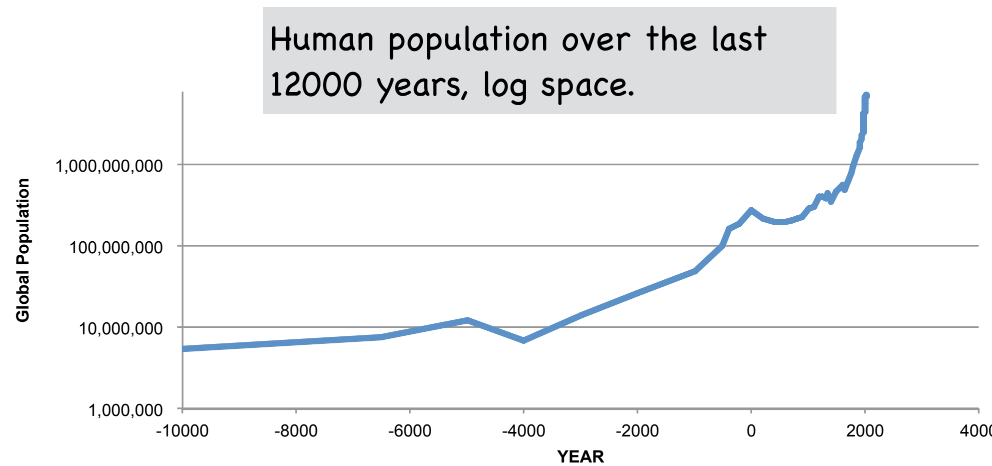
Unlimited, slowing slowly

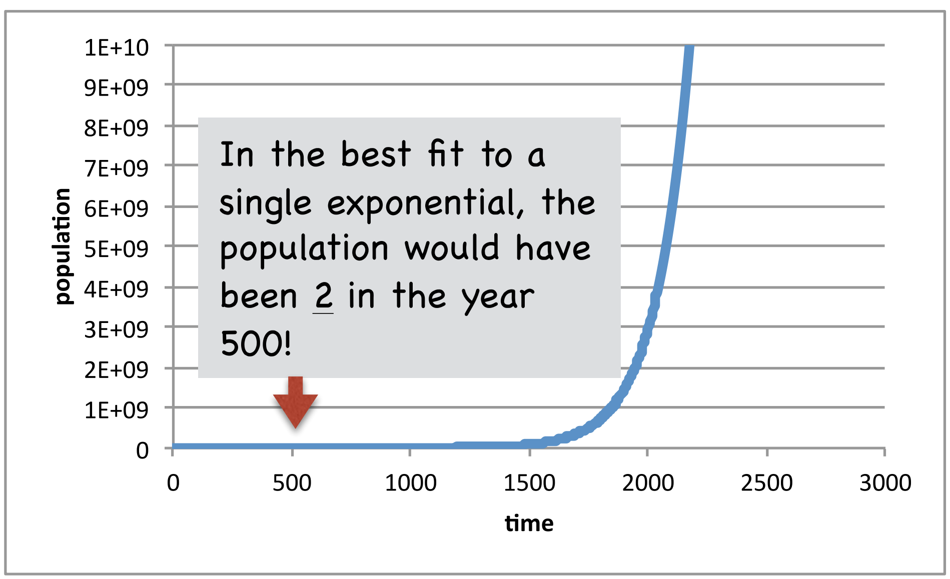


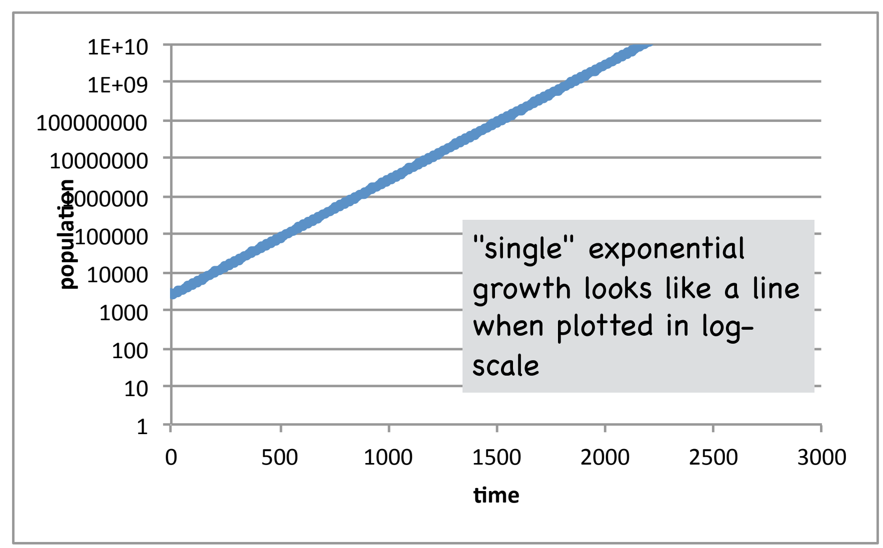
$$N_{t+1} = N_t + 1/N_t$$

Human population growth is not a "single-exponential"





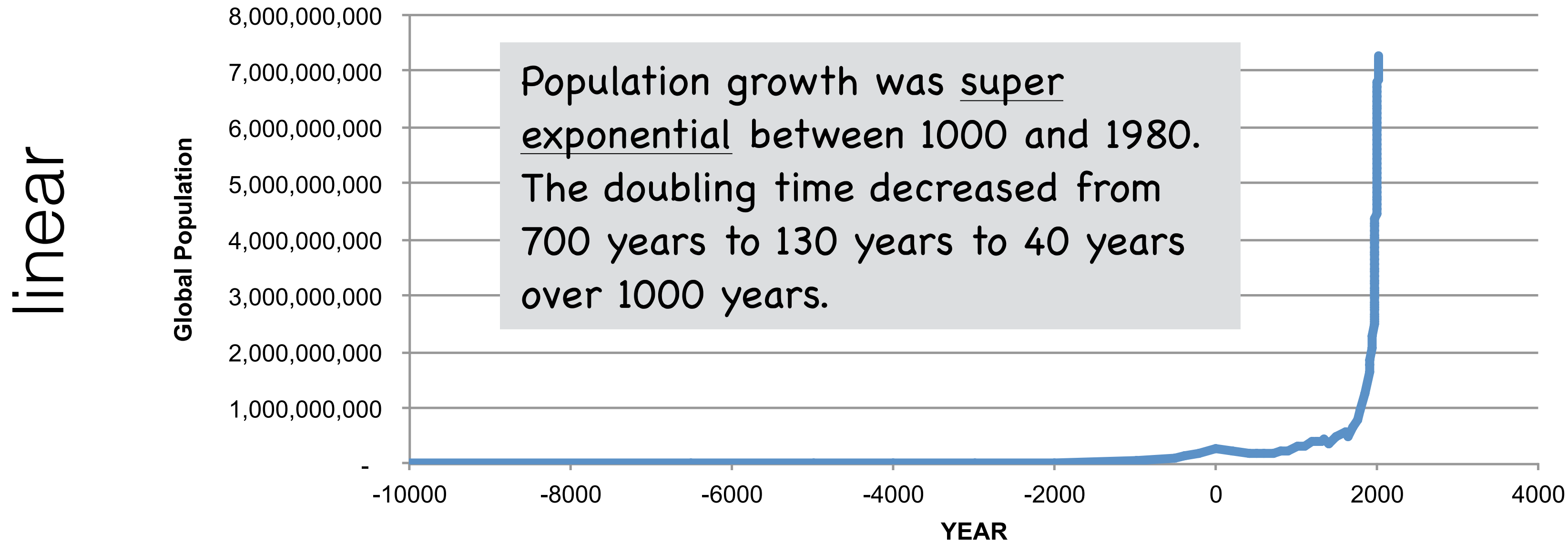




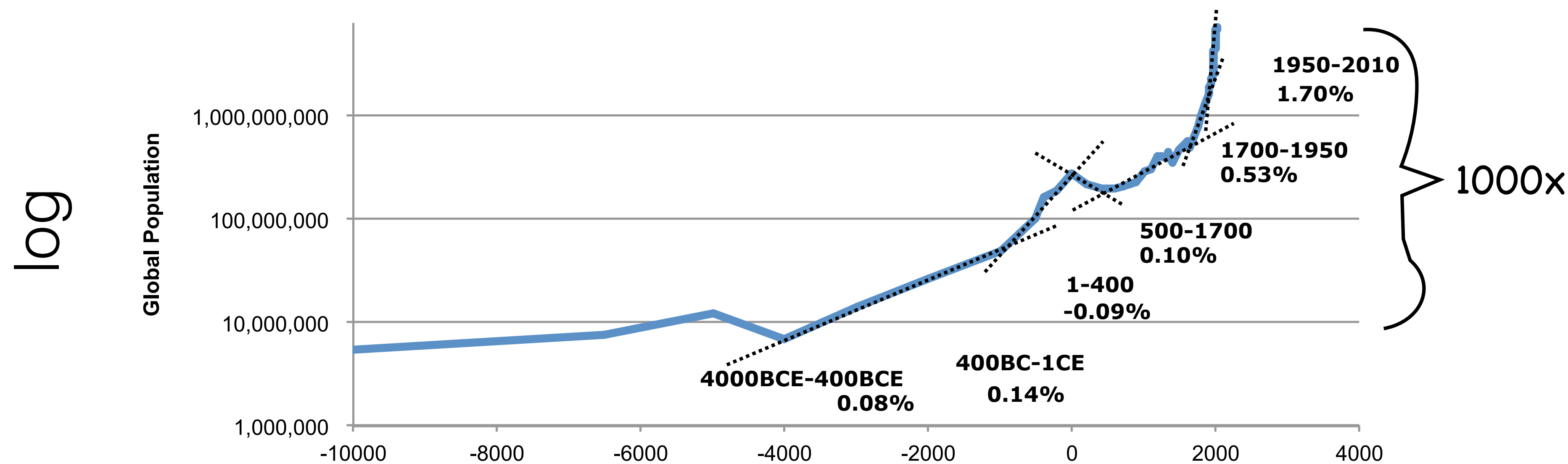
linear scale plots

log scale plots

The past 12 thousand years



Log-linear segments show periods of constant exponential growth/decline.



https://www.census.gov/programs-surveys/international-programs.html

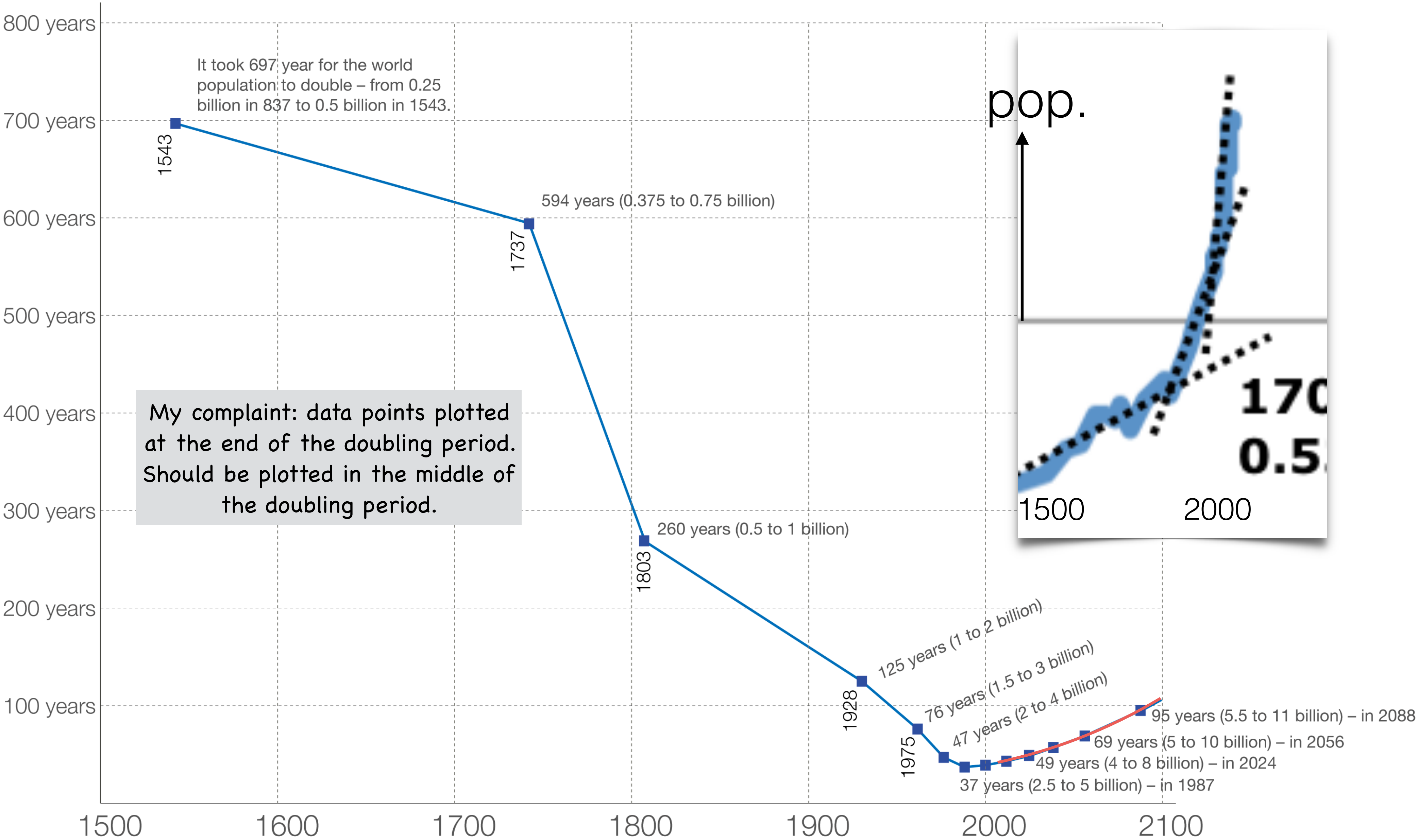
Why did population grow erratically?

Possible answers

1. Disease.
2. Technology.
3. Climate.
4. Disasters.
5. Regional differences.
6. Competition.

Time it took for the world population to double

Historical estimates of the world population until 2015 – and UN projections until 2100

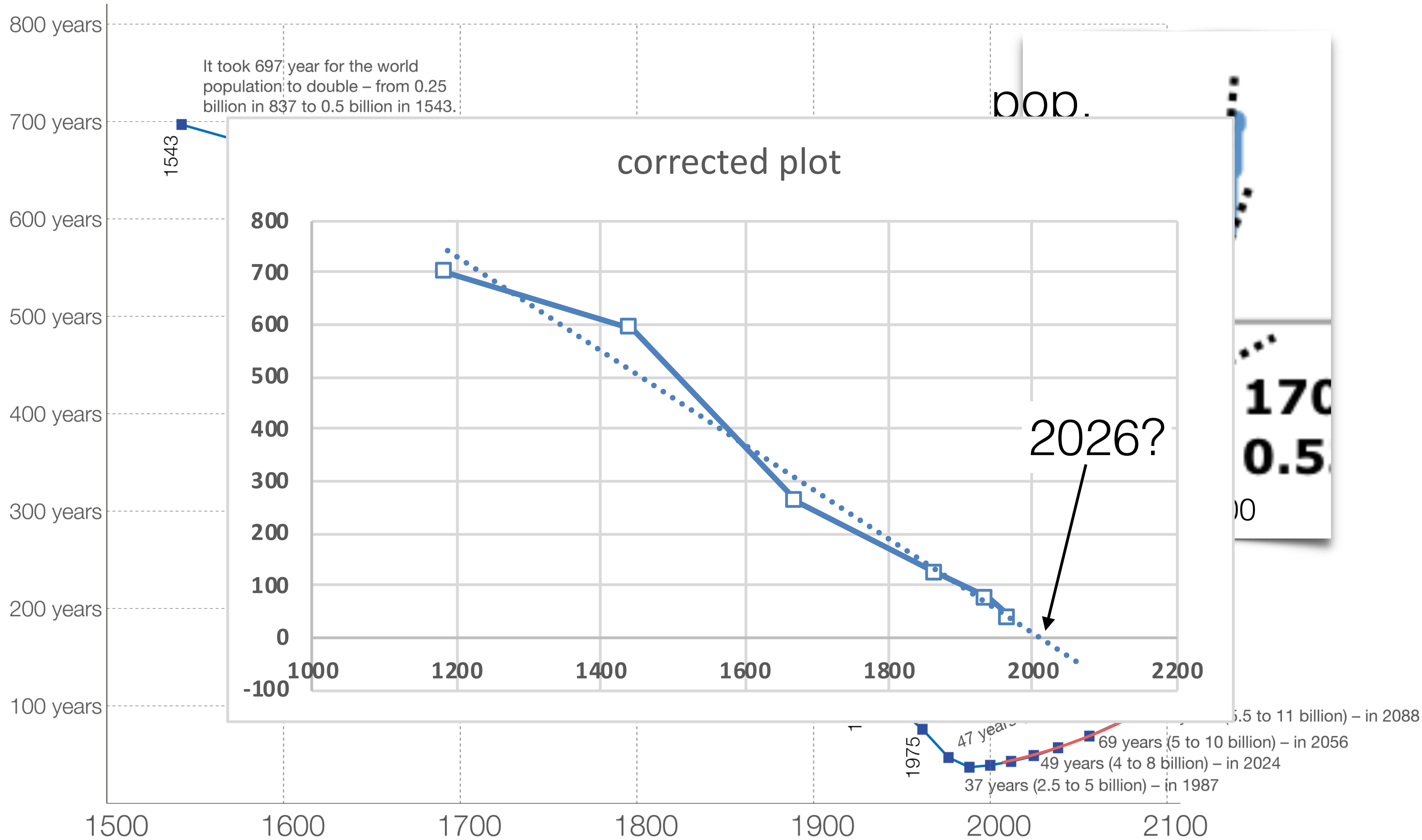


Data source: OurWorldInData annual world population series (Based on HYDE and UN until 2015. And projections from the UN after 2015 ('Medium Variant' 2015 Revision).
The data visualization is available at OurWorldinData.org. There you find the raw data, more visualizations, and research on this topic.

Time it took for the world population to double

Historical estimates of the world population until 2015 – and UN projections until 2100

Our World in Data



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The data visualization is available at OurWorldinData.org. There you find the raw data, more visualizations, and research on this topic.

~~Why~~ Did population grow erratically?

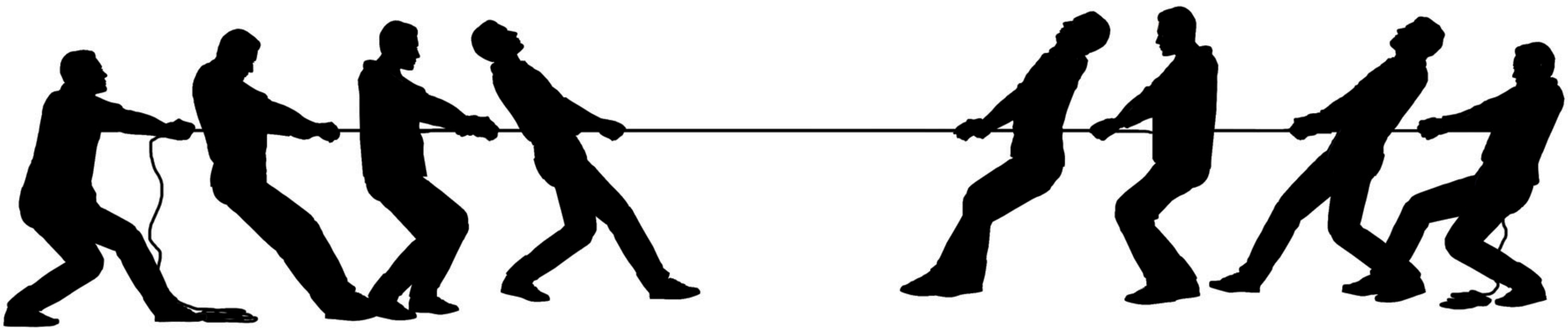
Possible points

1. More or less constant changes in the doubling time over 1000 years.
2. No one event (except 1350?) was large enough to make a dent in the global number.
3. Migration.

Curve fitting: Keep It Simple

KIS

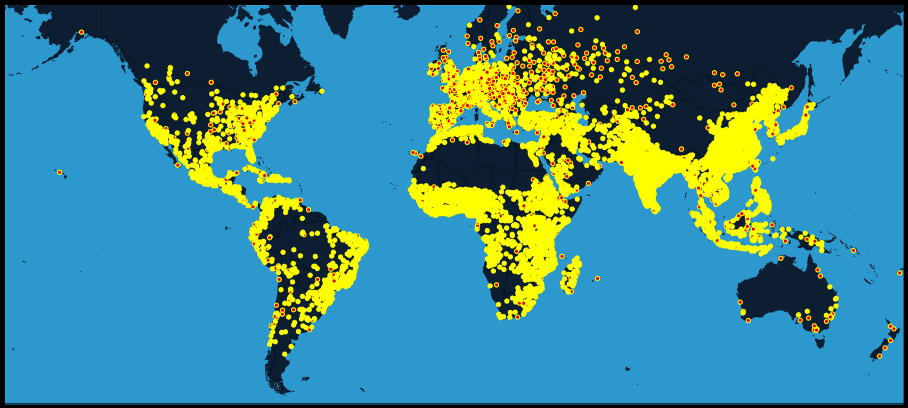
better fit



Number of parameters

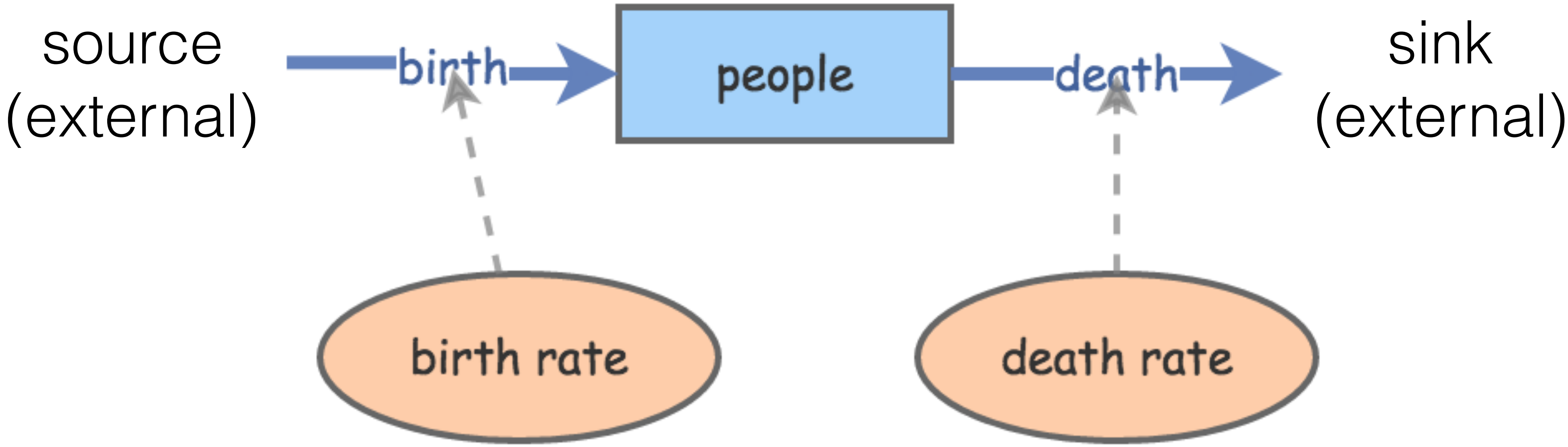
What sort of historical events correlate with increase/decrease in growth rate?

http://worldpopulationhistory.org/



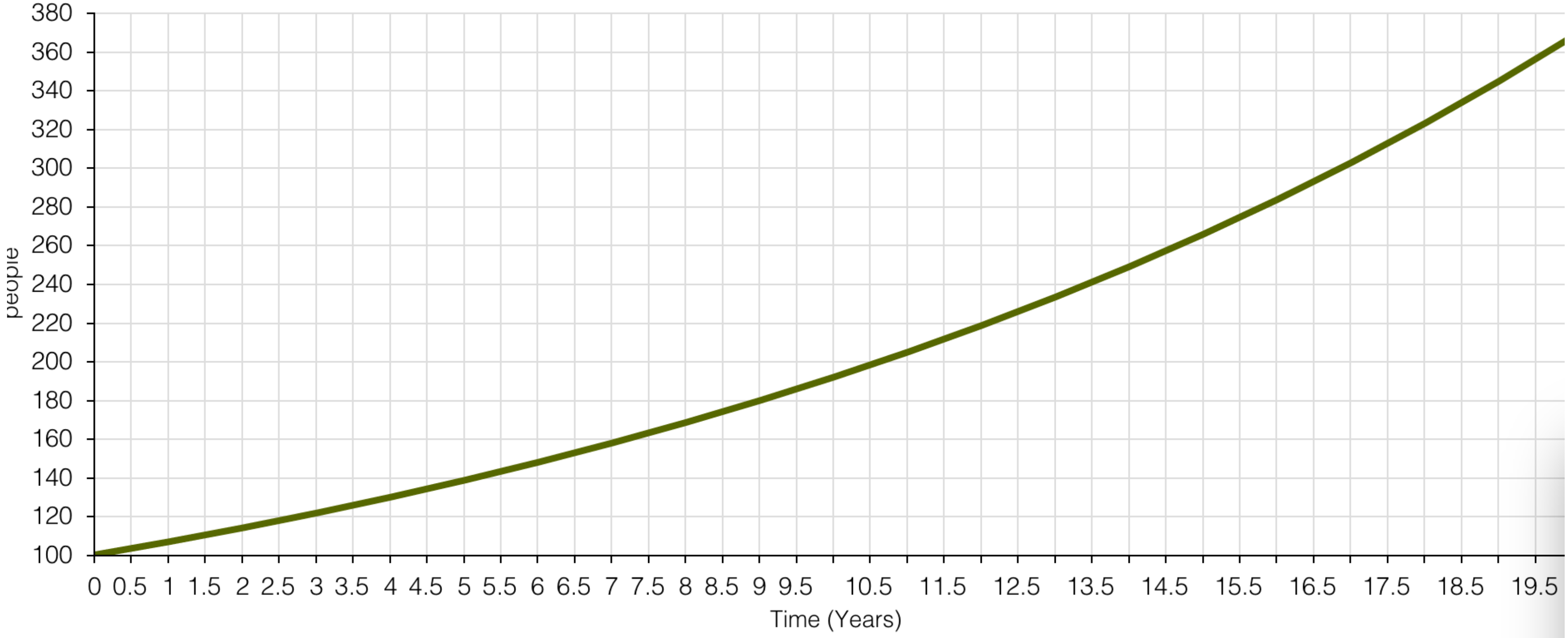
InsightMaker exercise

Part 1 : make a model for population



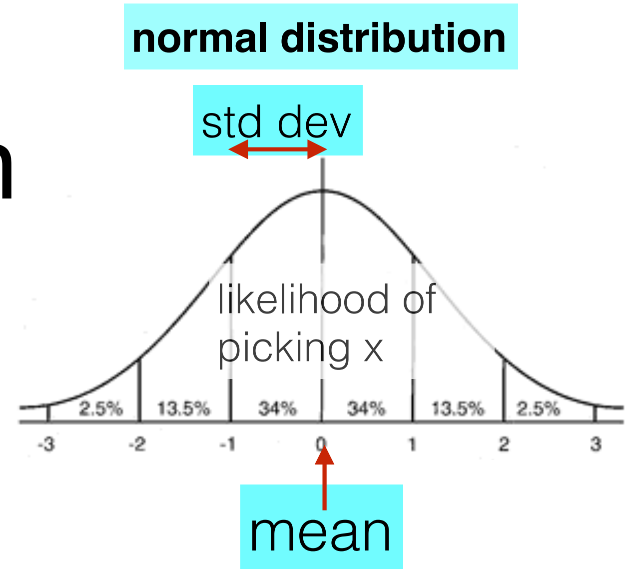
people = 100
birth = [birth rate]*[people]
death = [death rate]*[people]
birth rate = slider from 0 to 0.1
death rate = slider from 0 to 0.1

Results of a simulation



What if we want "error bars"?

Sensitivity testing in InsightMaker



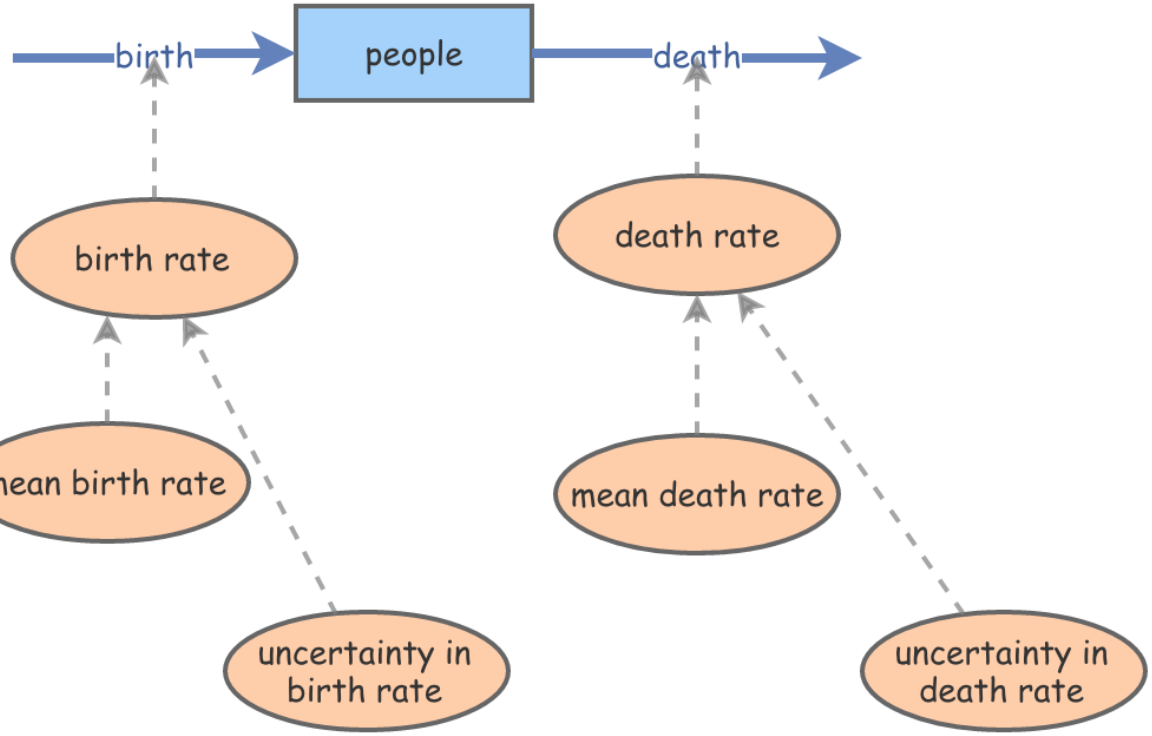
- *When the exact value of a variable is not known we test a distribution of values.*
- Must add a random number generator in = window (value/equation window)
  - value menu > Random Number Functions > Normal Distribution
    Places RandNormal(Mean,Standard Deviation) in equation window. Randomizes every timestep, unless "Fix" is added, then only randomizes on first timestep.
  - Fix(RandNormal(Mean,Standard Deviation))
  - Replace "Mean" with a number or Variable.
  - Replace "Standard Deviation" with a number or variable.

– Tools>Sensitivity Testing

  – Set Primitives to monitor, number of runs, confidence regions to plot. **Run analysis**.

in class exercise

IM 2 : sensitivity of a model for population



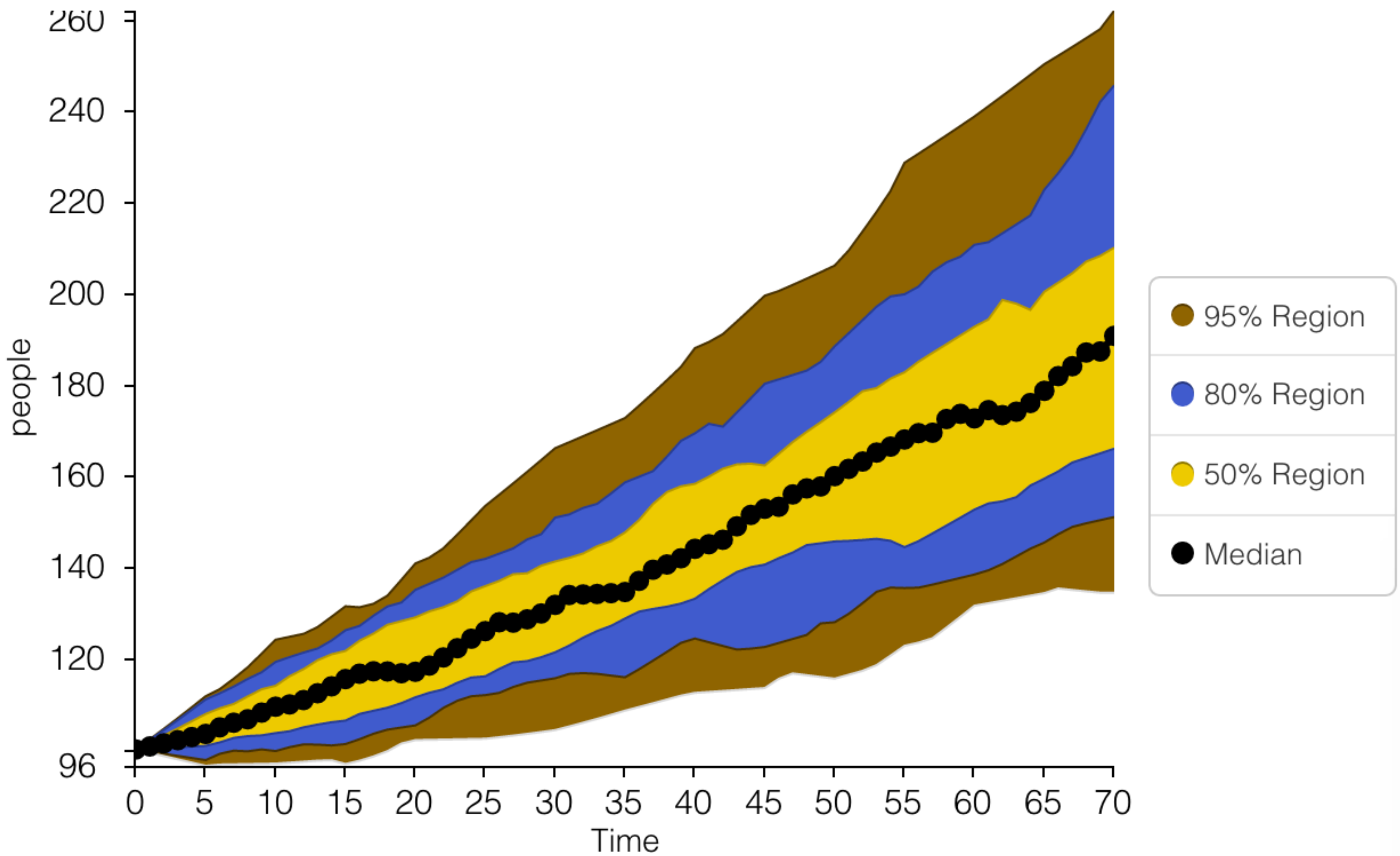
people = 100

birth rate = Fix(RandNormal([mean birth rate], [uncertainty in birth rate]))

death rate = Fix(RandNormal([mean death rate], [uncertainty in death rate]))

other variables are sliders.

Sensitivity analysis

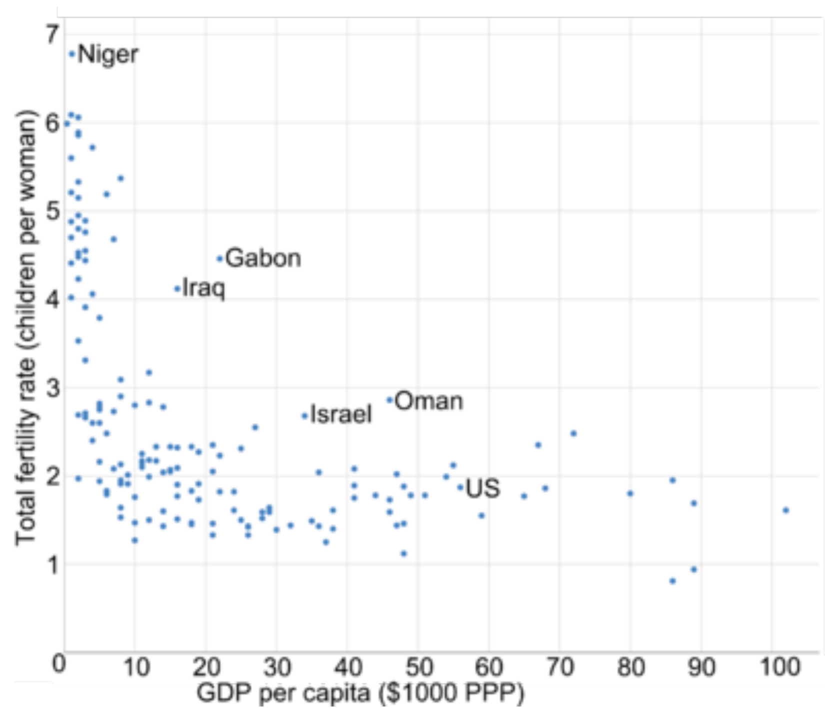


This says "my estimates of error in Variables A and B translate to this estimate of error in Stock Y (people).

Fine, but....

- How do we know the birth rate?
- How do we know how it will change?

Discuss LtG Figure 2-7 p.35



Next time.

- Demographics
- Read LtG pp 37-50
- Think about "cycle of poverty".